

# Towards the ideals of Self-Recovery and Metadata Privacy in Social Vault Recovery

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**Abstract**—Social key recovery mechanisms enable users to recover their vaults with the help of trusted contacts, or *trustees*, avoiding the need for a single point of trust or memorizing complex strings. However, existing mechanisms overlook the memorability demands on users for recovery, such as the need to recall a threshold number of trustees. Therefore, we first formalize the notion of *recovery metadata* in the context of social key recovery, illustrating the tradeoff between easing the burden of memorizing the metadata and maintaining metadata privacy. We present Apollo, the first framework that addresses this tradeoff by distributing indistinguishable data within a user’s social circle, where trustees hold relevant data and non-trustees store random data. Apollo eliminates the need to memorize recovery metadata since a user eventually gathers sufficient data from her social circle for recovery. Due to indistinguishability, Apollo protects metadata privacy by forming an anonymity set that hides the trustees among non-trustees. To make the anonymity set scalable, Apollo proposes a novel *multi-layered secret sharing* scheme that mitigates the overhead due to the random data distributed among non-trustees. Finally, we provide a prototype implementation of Apollo and report on its performance. Apollo reduces the chances of malicious recovery to between 0.005% and 1.8%, depending on the adversary’s ability to compromise. The multi-layered design shows a latency reduction from  $1.1 \times$  to  $740k \times$  compared to a single-layered approach, depending on the number of reconnections.

## 1. Introduction

Vaults, such as password managers [1], [2], [3], OS-managed storage services [4], [5], cryptocurrency wallets [6], [7], [8], play a key role in safeguarding digital assets. To maintain the secrecy of digital assets, vault applications encrypt their content with a key which is solely stored on the user’s device(s). However, if a user would lose her devices, she loses access to all her digital assets, which can be catastrophic [9], [10], [11]. To ensure guaranteed access to digital assets, vaults must incorporate a *recovery* mechanism.

Existing vault recovery solutions fall into two categories: (i) *custodial solutions*, where a trusted third party assists with recovery, and (ii) *self-sovereign solutions*, where the vault owner manages her vault’s recovery. While custodial solutions offer convenience, they introduce a single point of trust since a single external party holds the vault’s key and

assets. In contrast, self-sovereign approaches eliminate the single point of trust, however, require users to remember long alphanumeric strings, such as passwords or mnemonic phrases [7], [12], [13]. Hence, there exists a tradeoff—custodial solutions are convenient but need a centralized trust assumption whereas, self-sovereign solutions avoid centralized trust assumption, however place a significant memorability burden on users.

*Social key recovery* is a promising approach that aims to address this trade-off by enabling users to recover a vault’s key with the help of trusted contacts, or *trustees*. Social key recovery based vaults split the vault’s key into *shares*, which are distributed among trustees [14], [15], [16], [17], [18]. For recovery, the user can reconstruct the key by obtaining a predefined threshold number of shares. Therefore, social key recovery, in principle, appears to be an adequate solution for vault recovery as it eliminates the need to memorize complex strings, and removes the reliance on a single external party.

However, existing social key recovery mechanisms impose a cognitive load on users by implicitly requiring them to remember what we refer to as *recovery metadata* (Section 3), which includes: (i) vault existence; (ii) recovery procedure; (iii) trustees details; and (iv) threshold. Since device loss is unpredictable, it is unrealistic to expect users to recall recovery metadata when needed [19]. The need to reduce cognitive load leads to *self-recovery*, a property that ensures vault recovery even if a user forgets recovery metadata.

Consider the following thought experiment to illustrate the ideal of self-recovery. Due to a traumatic event, Alice suffers from amnesia, forgetting not only her identity but also the existence of her social connections and the vault containing her assets and identity documents. Over time, a few former friends reconnect with her, providing enough information and possibly physical keys to the vault, that enables her to rediscover and recover her identity and assets. This scenario exemplifies “self-recovery” as Alice does not need to remember any details to retrieve her vault.

The scenario of vault recovery outlined represents an ideal that is difficult to achieve; our goal is to facilitate recovery when the user forgets the recovery metadata. A trivial approach of publishing metadata in a public blockchain would achieve self-recovery for us, however, it would undermine privacy. In particular, such a design leaks information about users’ closeness and presents the metadata to malicious parties, who can extort shares from trustees.

Therefore, the recovery metadata needs to be kept private, and this elicits the concept of *recovery metadata privacy*. Recovery metadata privacy can be trivially achieved by having the user remember the metadata and not store it elsewhere. However, this approach sacrifices self-recovery. An alternative for maintaining privacy is encrypting the metadata before publishing, but this requires a user to retain the encryption key even after device loss—leading to a recursive key recovery problem. This highlights the trade-off between *self-recovery* and *recovery metadata privacy*, posing a fundamental question:

*Is it possible to enable vault recovery even if a user forgets the recovery metadata, while protecting the metadata privacy?*

In this paper, we answer this question affirmatively, and introduce Apollo, the first social key recovery framework that concurrently achieves both *self-recovery* and *recovery metadata privacy*. Fig. 1 depicts Apollo’s high-level design, which is based on two key observations: (i) a user will naturally reconnect with people she already knows with minimal effort [20], [21]; and (ii) while a user can acquire shares by reconnecting with contacts, an adversary must actively seek them out, requiring more effort than the user. Apollo leverages the distributed trust of a user’s entire social circle—in addition to backing up shares of the key with trustees, Apollo distributes randomized data among the rest of the people, the *non-trustees*, such that trustees and non-trustees are indistinguishable. A user does not need to put dedicated effort for recovery since she obtains data from everyone she reconnects with. She only needs to remember her social ties vaguely, which is less probable to be forgotten and more relaxed than remembering the exact recovery metadata. In contrast, an adversary needs to extort enough relevant shares while the trustees are hidden. Thus, distributing indistinguishable data provides three main benefits: (i) the user does not need to remember the recovery metadata since she will eventually acquire enough shares by reconnecting; (ii) indistinguishability conceals trustees among non-trustees, which protects users’ personal choices; and (iii) since indistinguishability increases the number of potential trustees, an adversary is forced to compromise more people to get enough relevant data. This increases the cumulative effort required from an adversary and the probability of the adversary getting reported, making malicious recovery an overwhelming task.

While indistinguishability enables self-recovery while preserving metadata privacy (Section 5), it adds a computation overhead during recovery. Since key shares and random shares appear identical, the recovery algorithm has to brute-force all possible share combinations and the number of combinations grows exponentially (the combinatorial operator— $\binom{n}{r}$  is exponential in  $r$ ). We address this by introducing *multi-layered secret sharing* (MLSS), a novel approach that structures shares into two layers: (i) the first layer consists of additive shares of the key; (ii) the second layer contains shares of the first layer, structured as Shamir’s secret shares with a fixed threshold. This design ensures

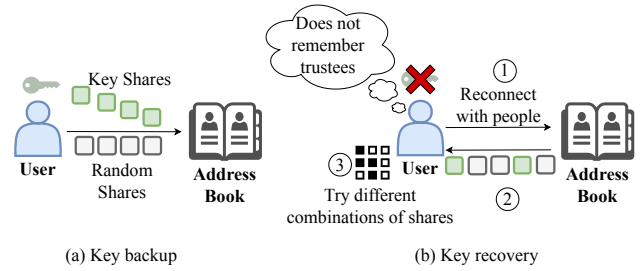


Figure 1. Overview of working of Apollo.

that the number of possible combinations remains small and manageable, sidestepping the exponential growth issue. Thus, Apollo achieves scalability without compromising self-recovery or recovery metadata privacy.

We implemented Apollo in Go [22] and evaluate the performance of the recovery. Apollo provides an exponential saving in the running time of the recovery algorithm, and we observe factor ranging from  $1.1 \times -740k \times$  over a naive single-layered approach. For instance, after reconnecting with 30 people, Apollo takes only 0.12 seconds for computing over all combinations, while the single layered approach takes more than 25 hours. In addition, we empirically analyze Apollo’s security against an adversary trying to recover by extorting shares. Apollo reduces the likelihood of malicious recovery to 0.005% – 1.8%, with reasonable assumptions for adversary’s capabilities.

This paper makes the following novel contributions:

- A taxonomy of *recovery metadata* in social key recovery that motivates *self-recovery*, and the need for *recovery metadata privacy* (Section 3).
- The design of *Apollo*, the first ever key recovery method that concurrently achieves self-recovery and recovery metadata privacy (Sections 4 and 5).
- The *multi-layered secret sharing* (MLSS) scheme that reduces recovery computation time to make Apollo’s adoption practical (Section 5.4).
- A prototype implementation of Apollo along with its performance and empirical security analysis (Section 6).

## 1.1. Goals and Limitations

**Goals.** Apollo aims to achieve the following goals.

- *Low memorability:* A user can recover her key without remembering the exact recovery metadata (e.g., trustees).
- *Confidentiality:* The backed-up information does not leak any recovery related details (e.g., trustee status).
- *Unauthorized access mitigation:* An adversary, trying to recover a user’s key by extorting backed up data, fails with high probability.
- *Efficiency:* The recovery algorithm executes within a few minutes, even for large address books.

**Limitations.** Apollo does not address the following:

- 1) Secure storage, availability and vaults management. Refer to [4], [12], [13] for details on these aspects of vaults.

- 2) Adversarial ability to extort the key from the user itself. This issue is orthogonal to Apollo, and has been extensively studied in e-voting literature [23], [24], [25], [26].
- 3) Apollo does not ensure data integrity, allowing an adversary to store false information with contacts to hinder a user’s chances of recovering her key. However, this attack is unlikely as it requires compromising many people.

## 2. Background and Motivation

This section outlines a classification of existing vaults along with their recovery strategies (Section 2.1), and the challenges that motivate Apollo (Section 2.2). We discuss other related work in Section 8.

### 2.1. Background

**Custodial Vaults.** In custodial vaults, such as file hosting services [4], [5], [27], [28] centralized cryptocurrency wallets like Coinbase [6], and digital identity storage services like Redsafe [29], the service provider holds the key for encrypting a vault’s contents. These solutions guarantee restoring access to the vault, even if a user loses all her devices. Access restoration is often based on two-factor authentication (via email or phone call), which does not incur a heavy cognitive load on users. However, such solutions pose the risk of a single point of trust, as the assets and the keys encrypting them are held by the service provider.

**Self-sovereign Vaults.** Self-sovereign vaults, such as Keybase [12], Keeper [13], hardware cryptocurrency wallets [8], [30], [31], and wallets with mnemonic phrases [7], eliminate the single point of trust by letting users manage their vault’s key. However, these solutions demand more memorability requirements from users, compared to custodial vaults. Existing self-sovereign vaults, such as Keybase [12] and Keeper [13], expect users to download and store their secret keys. Hardware wallets fail if all devices are stolen and mnemonic phrases are hard to remember. Thus, despite obviating the need for a trusted-third party, the adoption of self-sovereign solution is limited because of the burden they impose on users.

**Social Key Recovery.** To improve the practicality of self-sovereign solutions, social key recovery-based approaches [14], [15], [16], [32], [33] have been deployed. A user chooses a set of trusted people, referred to as *trustees*, and distributes the *shares* of her key among them. These shares are typically Shamir’s secret shares [34] and, a user can recover her key if she obtains a threshold of them.

Social key recovery based solutions are considered to address the limitations of existing self-sovereign approaches since they do not require users to memorize complex strings. However, if a user fails to remember some recovery related data, such as the trustees details, then she would fail to recover the key. The memorability burden imposed by existing social key recovery mechanisms remains unexplored, and thus, minimizing the burden in social key recovery mechanisms still remains an open research problem. In this paper, we address this problem, and close this research gap.

### 2.2. Motivation

Custodial solutions offer a convenient means for key recovery but rely on a single point of trust. Self-sovereign protocols eliminate the single point of trust, though they place a significant memorability burden [7], [13]. Social key recovery mechanisms distribute the key among a set of trustees aiming to reduce the memorability burden as compared to the typical self-sovereign counterparts. Not addressing the memorability issue would require users to identify their trustees by asking their contacts if they hold any shares. Since sharing secrets with people is representative of interpersonal trust, non-trustees might feel slighted to learn that the user did not trust them enough. Thus, neglecting relationship harm would force users to back up in a “socially acceptable” manner, which undermines *users’ control*—a core principle of self-sovereign design.

Naive extensions of social key recovery based approaches to prevent the damage of interpersonal relationships are sub-optimal. For instance, a user can share the key shares among everyone she knows (hence everyone in the social circle is a trustee) and configure the threshold accordingly. However, such an approach restricts the user’s autonomy in choosing her trustees set, defeating the goal of self-sovereignty in social key recovery.

None of the previous work has addressed the memorability issue in social key recovery approaches or the social sensitivity of the choice of trustee set. Thus, in the next section, we first discuss the memorability requirements enforced by social key recovery mechanisms and provide a novel taxonomy of *recovery metadata*.

## 3. Recovery Metadata Management Problem

This section describes recovery metadata management (Section 3.1), illustrating the need for self-recovery (Section 3.2) and recovery metadata privacy (Section 3.3).

### 3.1. Recovery Metadata

Recovery metadata represents the data that a user is expected to remember for vault recovery. In this work, we solely focus on the recovery metadata related to social key recovery based solutions and the extreme scenario where a user loses her key due to device loss. To our knowledge, our work is the first to thoroughly assess the notion of recovery metadata. Existing solutions [14], [15], [16], [17], [18], [33], [35] implicitly require users to remember the metadata. Therefore, we first outline the four main recovery metadata that the existing solutions expect users to remember, and analyze the drawbacks of this unrealistic expectation.

*RMI: Vault Existence.* Vault existence represents the fact that the user had setup a vault and all existing solution that a user would remember this metadata. However, this assumption would fail if a user had setup a vault a long time ago [19]. For instance, people can forget about setting up a password manager on one of their old laptops.

*RM2: Recovery Procedure.* Recovery procedure corresponds to how a user must proceed with recovery. In particular, a user must proceed with contacting her trustees and Prior works expect users to remember the recovery procedure until the day of device loss and this assumption would fail since users may not be able to recall the procedure after some time [19].

*RM3: Trustee Details.* This metadata represents the people that a user should contact for recovery. Existing vault mechanisms either expect users to remember trustees or they have a small number of trustees to keep the list short (e.g., five in Apple’s recovery contacts). The former design is not inadequate as people can forget trustees [19], while the latter constrains users to choose a small number of trustees (e.g., Apple’s recovery contacts) and does not address the memorability problem as depicted by Schechter et al. [36]—where users failed to remember *3-out-of-4* trustees.

*RM4: Parameters (Threshold).* The recovery procedure may require users to remember additional parameters; in our case, we focus on the threshold. A user cannot remember the threshold (a number) for a long duration [19]. Some existing solutions eliminate the need to remember by fixing the threshold (e.g., three in Apple’s recovery contacts). However, fixing the threshold compromises on user autonomy, the central premise of self-sovereign approaches.

Table 1 depicts the assumptions made by existing solutions for remembering recovery metadata. Most assume that users remember the entire recovery metadata, except for a few that do not require memorizing the threshold—they use fixed percentage threshold [14], [15], or a fixed configuration [33], [35]. Hence, with minor memorability issues, recovery using existing solutions becomes an arduous task. It would be desirable if a recovery mechanism could minimize the amount of metadata to be remembered, or ideally, even make recovery possible without remembering metadata. This brings us to the notion of *self-recovery*.

Protocol	Metadata				Privacy
	Existence	Procedure	Trustees	Threshold	
Argent* [14]	×	×	×	✓	✓
Loopring* [15]	×	×	×	✓	✓
DeRec Alliance [17]	×	×	×	×	✓
Indy [16]	×	×	×	×	✓
Recovery Contacts <sup>†</sup> [33]	×	×	×	✓	✓
HTC EXODUS 1 <sup>†</sup> [35]	×	×	×	✓	✓
Social wallet [18]	×	×	×	×	✓
Self-recovery (Section 3.2)	✓	✓	✓	✓	×
“Privacy-preserving” recovery (Section 3.3)	×	×	×	×	✓
<b>Apollo</b> (Section 5)	✓	✓	✓	✓	✓

\* - fixed percentage threshold of  $> 50$ ; †- fixed configuration (3-out-of-5).

TABLE 1. TABLE DEPICTING IF THE MEMORABILITY AND PRIVACY OF RECOVERY METADATA IS CONSIDERED IN VARIOUS SOLUTIONS; SOLUTIONS BETWEEN THE DASHED LINES ARE IDEAL SOLUTIONS FOR MEMORABILITY MINIMIZATION AND METADATA PRIVACY.

### 3.2. Self-recovery

Self-recovery represents the ideal vault recovery design, where a user can recover her vault without remembering any of the metadata. Self-recovery can be made possible by publishing the entire metadata (e.g., on a public blockchain). With such a design, a user can easily access the recovery metadata, and proceed with the vault’s recovery. However, such a design is a privacy nightmare as it not only makes the key vulnerable but also leaks information about the user’s personal life. The need for safeguarding the recovery metadata leads to the concept of *recovery metadata privacy*.

### 3.3. Recovery Metadata Privacy

Recovery metadata privacy ensures that no one learns anything about the metadata. This is essential since the leakage of metadata could lead to security and privacy risks. If we publish the trustees details, then everyone would learn a user’s precise *closeness* with others. On learning the trustees details, an adversary might start phishing or bribing trustees to recover the vault. Hence, recovery metadata needs to be kept private to prevent unwanted leakage about one’s personal life and to mitigate the chances of an adversary recovering the key. Prior works have overlooked this requirement, and do not inform users to keep the recovery metadata confidential. Furthermore, prior works have never considered the possibility of a vault being maliciously recovered because of trustees getting compromised, and leave it up to the user to choose the users to choose an *adequate* trustee set for secure backup. Next, we describe the reasons why recovery metadata privacy is a necessity.

*RM.PR1: Preserving interpersonal relationship.* If we try achieving self-recovery by publishing information, then we leak the trustees status to the non-trustees. This could potentially harm the user’s relationship with non-trustees as they might be disappointed to learn that the user does not consider them *close enough* [37].

*RM.PR2: Exercising freedom in backup.* If a user tries to not harm her social ties while publishing metadata for self-recovery, she would choose a *socially acceptable* trustees set [38]. This deprives the user of her freedom to choose her trustees, and could risk her recovery chances, e.g., backing up data with a tech-averse relative. Furthermore, a user might be judged for changing some of her trustees to non-trustees or vice-versa, thus preventing a user from changing her trustee set based on her personal trust levels.

*RM.PR3: Security.* If recovery metadata is published, then it would reveal the route to recovering user’s vault to unrelated third-parties. This makes a user’s vault vulnerable to unauthorized accesses since an adversary can recover the key by extorting shares.

**“Privacy-preserving” recovery.** Achieving only recovery metadata privacy is simple—we just need the user to memorize the entire recovery metadata, and not store it elsewhere (like prior solutions in Table 1). However, this naive design fails since it expects users to memorize a

lot of information, which is unreasonable as people forget information with time [19], [19], [39]. Due to the fear of forgetting trustees, users might choose fewer trustees. Furthermore, having the users remember their recovery metadata does not ensure freedom in backup—a user cannot change her trustees set with freedom.

**Trustee privacy.** These challenges highlight a critical property for achieving recovery metadata privacy: in addition to hiding recovery metadata, no one other than the user should know about the trustee status of contacts. That is, even trustees should remain oblivious to their trustee status. We refer to this property as *trustee privacy*, which is essential for letting a user exercise her freedom in setting up her backup, a necessity for any self-sovereign vault.

### 3.4. Ideal Recovery: Self-Recovery with Recovery Metadata Privacy

Korir et al. [40] highlight that users consider recovery to be a *fundamental feature that should be automated*. Hence, self-recovery is a must in social key recovery solutions, or at least, system designers should consider minimizing the amount of metadata that a user should remember for recovery. On the contrary, recovery metadata privacy is a necessity if we are aiming to provide user autonomy and protection against unauthorized accesses. However, achieving both self-recovery and metadata privacy simultaneously is a challenging yet essential task, as each requires opposing approaches. Hiding information enhances privacy but burdens the user to remember the hidden information, while publishing some metadata favors self-recovery but compromises privacy. Therefore, social key recovery solutions inherently face a trade-off of recovery metadata memorability minimization and recovery metadata privacy. Apollo takes the first step towards building a vault recovery mechanism that provides self-recovery as well as protects metadata privacy with reasonable assumptions.

## 4. Overview of Apollo

This section presents Apollo’s design at a high level, covering system model (Section 4.1), assumptions (Section 4.2), threat model (Section 4.3), and design components enabling self-recovery with metadata privacy (Section 4.4).

### 4.1. System and Communication Model

**System Model.** There are four main components of Apollo: (i) *user*; (ii) *vault*; (iii) *vault’s key* ( $\kappa$ ); and (iv) *address book* of the user (contacts). We focus on the backup and recovery of  $\kappa$  and refer to [4], [12], [13] for other aspects of vaults. We assume that Apollo is synchronized with the address book for assisting the user in backing up  $\kappa$ . Apollo is not a part of address book software as users usually back up their address book with cloud services, and we do not want information related to  $\kappa$  to be stored on cloud. We envision Apollo to be used as a separate application,

and the contacts’ devices to store backed up information locally. Apollo divides the address book into: *trustees*, the people who hold relevant recovery data, and *non-trustees*, the contacts who are not trustees.

$\kappa$  is a bit string of variable length and can be represented as an array of elements of binary extension field of degree  $n$ , i.e.  $\kappa \in \mathbb{F}_n^* = GF(2^n)$ . Apollo uses Shamir’s secret sharing [34] to generate shares of  $\kappa$ . Apollo needs a threshold  $\tau$ , the fraction of trustees needed for recovery, which is set by the user. The user can manually update the set of trustees, non-trustees, and the threshold.

When a user loses her device and thus, her vault’s key, she reconnects with her contacts using a new device: user and contacts exchange and store each other’s details (e.g., phone number) on their devices. For the core design of Apollo’s, we assume that contacts’ devices are responsive, store the shared data, and the user’s new device reliably obtains blobs from these contacts after reconnecting.

**Communication Model.** We assume that the user’s device and the contacts’ devices are connected over an asynchronous network, allowing them to communicate over an end-to-end encrypted channel, used for exchanging packets during key backup and key recovery. A user reconnects with her contacts using an authenticated out-of-band channel, and proves her identity to them during recovery. For example, a user can meet someone in-person or connect via a phone call with the help of another friend.

### 4.2. System Assumptions

Apollo relies on the following assumptions.

- 1) Users and their contacts own a personal device (e.g., smartphone) with the Apollo application installed.
- 2) Apollo is managed by the device’s OS to leverage communication details like message logs and emails. Although not a necessity for Apollo’s functioning, this assumption eliminates the need for installation of a separate application.
- 3) We assume that a user vaguely recalls her closeness with others due to which she connects within her social circle. The user does not remember the exact recovery metadata.

### 4.3. Threat model

We assume that contacts’ devices reliably store the recovery information, do not tamper with the data, and do not interfere with the packet exchange process. We assume that an adversary knows about the user’s social circle (contacts) but not as well as the user does. We model the assumption using percentage error in pinpointing trustees. We assume that the adversary’s error rate is  $k_{\text{err}}$  times more than user’s error due to forgetting. The adversary can compromise contacts to extort information and she can use methods such as phishing, bribery or coercion or any other means for extortion. We assume that the probability of the adversary extorting shares is less than the user’s probability of obtaining them and hence, we assume the probability of

extortion to be  $k_{\text{obt}}$  times less than the user’s probability of acquiring shares. Finally, we assume that an adversary carries a risk of getting reported while extorting people, and we assume that reporting leads to the user changing the key or the adversary being caught, thereby halting the malicious recovery attempt. We model reporting using a probability  $p_{\text{rpt}}$ . We do not provide precise value of  $k_{\text{err}}$ ,  $k_{\text{obt}}$  and  $p_{\text{rpt}}$  since the theoretical analysis of Apollo’s recovery probability is intractable. Instead, we analyze the security of Apollo empirically as a function of these parameters and provide a range for each later in Section 7.2. We leave defense against side channel attacks out of scope of this work.

#### 4.4. Design Overview

Apollo enables self-recovery while ensuring privacy of recovery metadata. Apollo distributes indistinguishable packets across the user’s social circle, represented by their address book in Apollo. The packets are distributed such that the trustees hold relevant data for recovery while the non-trustees hold random data and, any two packets are indistinguishable from each other. Since every contact holds some information, each can be a potential trustee, thus avoiding the risk of damaging relationships. In turn, users need not concern themselves with social conformity when selecting trustees. For recovery, the user obtains shares by simply reconnecting with people since every contact holds a share, and she eventually recovers her key after reconnecting with *enough* trustees, which we detail in Section 5. Apollo eliminates the need to remember recovery metadata since a user can recover as long as she wants to reconnect within her social circle. This is a reasonable assumption since (i) people naturally want to connect with others [20], [21]; and (ii) social connections are generally associated with our emotions, which are linked to our *episodic memory*, and the loss of episodic memory is slower as compared to other types of memories [41].

On the contrary, this design makes malicious recovery difficult for an adversary. Due to packet indistinguishability, everyone in a user’s social circle is potentially a trustee. Hence, an adversary would have to expend some effort to identify trustees and then, expend more effort to extort shares from them. In particular, an adversary would face some resistance while trying to extort shares from each contact and would increase her chances of getting reported with each extortion attempt. Therefore, Apollo forces an adversary to exert a significant amount of effort for obtaining *enough* relevant data, while increasing the chances of getting reported with each attempt—this makes malicious recovery a daunting task in Apollo.

Although indistinguishability helps Apollo achieve self-recovery along with metadata privacy, it does not address scalability—a recovery algorithm cannot distinguish between relevant and chaff packets. Hence, the recovery algorithm has to brute-force through all possible combinations to find relevant packets for recovery. The combinatorial operator,  $\binom{n}{r}$ , grows exponentially with  $r$ , where  $n$  and  $r$  represent the number of acquired shares and threshold respectively.

Thus, the number of combinations grows rapidly with the threshold, making recovery impractical. Apollo utilizes the novel *multi-layered secret sharing* (MLSS) scheme to reduce the recovery time by orders of magnitude. By using multiple levels of shares, MLSS sidesteps the need to try all possible combinations, which eliminates the exponential growth and makes the design of self-recovery along with metadata privacy practical.

In summary, Apollo provides: (i) self-recovery; (ii) recovery metadata privacy; (iii) protection against malicious recovery attempts; and (iv) a means to make this recovery paradigm practical.

### 5. Apollo Protocol Design

This section details Apollo’s design. First, we explain the key backup (Section 5.1) and the key recovery process (Section 5.2), the components of the system that a user interacts with. Then, we outline the multi-layered secret sharing scheme (Section 5.4), the protocol used in Apollo for efficient recovery.

#### 5.1. Backup

For backup, Apollo utilizes the user’s address book, representing all the people she knows. Apollo backs up *packets* such that trustees and non-trustees are indistinguishable. Packets in Apollo contain shares of the vault’s key and tags for indicating successful recovery. By distributing indistinguishable packets, Apollo achieves self-recovery while maintaining recovery metadata privacy. Fig. 2 illustrates the backup procedure. The numbers in the figure show the sequential steps, which we detail below.

**1) Preparation.** The backup process begins with the user installing Apollo on her device, and creating a vault with its key. Then, Apollo assists the user in utilizing her address book to choose her trustees. Apollo uses various information associated with the address book such as messages and call logs, to recommend a tentative trustees set to the user. The remaining contacts form non-trustees. For generating secret shares, Apollo needs a threshold—the fraction of trustees needed for recovery. To reduce the burden on users, Apollo sets a default threshold, although advanced users have the option to set this value themselves.

**2) Generation.** After configuring the address book and setting a threshold, Apollo proceeds to generate packets for key backup. Apollo uses Shamir’s secret sharing [34] to generate shares of the key for trustees (*key shares*) and it generates shares that are irrelevant to recovery (*random shares*) for non-trustees. Before distributing a share, Apollo appends a tag to create a *blob*. The tag is essential for verifying successful recovery and like random shares, the tag held by any non-trustee does not help with recovery and is only included to maintain indistinguishability. Each packet distributed to a contact is an array of these blobs.

**3) Distribution.** After packets are generated, user’s device sends packets to devices of contacts. On receiving a packet, a device sends an asynchronous acknowledgement

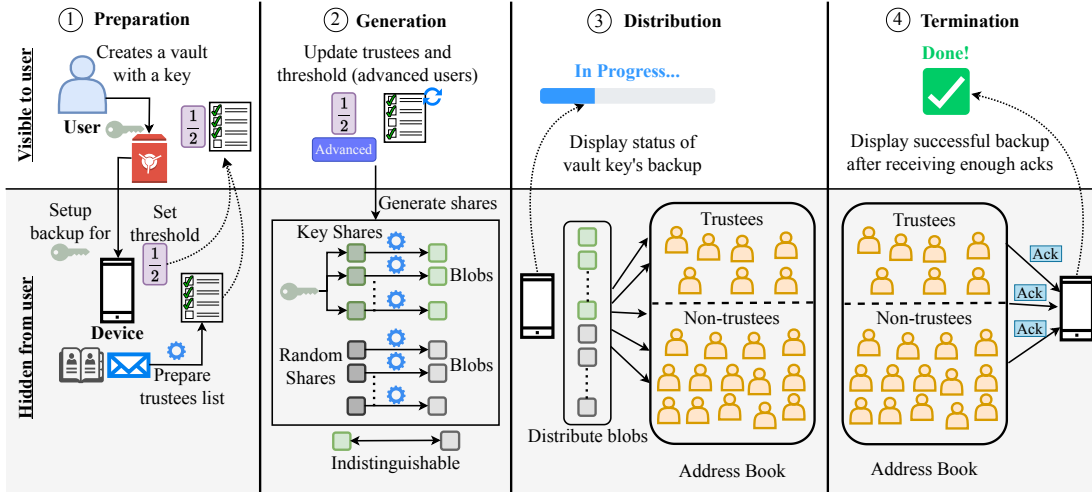


Figure 2. Key backup: figure does not include devices of contacts for clarity.

back to the user’s device. Using the acknowledgement, the Apollo application on the user’s device learns that the recovery information has been backed up by that contact. During this phase, user’s device provides the progress of backup based on the number of acknowledgements received.

**4) Termination.** After user’s devices receives acknowledgements from enough contacts the backup ends, and the device notifies user about the successful backup. If user’s device does not receive an acknowledgement from a contact after a threshold amount of time, then it re-sends the corresponding packet. For the core design of Apollo, we assume that every contact will be online at some point in near future, and thus, user’s device eventually receives all the acknowledgements. We discuss the case when this assumption does not hold in Section 7.

## 5.2. (Self-)Recovery

We assume the worst-case scenario where the user loses her device, and does not remember the recovery metadata (Section 3). First, she gets a new device and reconnects with people she knows for rebuilding her digital life. Apollo’s goal is to facilitate recovery in this setting. Fig. 3 depicts the key recovery process. The numbers in the figure show the sequential recovery steps, detailed below.

**1) Acquisition.** While reconnecting with people, the user and the contact exchange contact information with each other and then, the user’s device queries the reconnected person’s device for a packet. The contact’s device sends a packet to user’s new device. As the user continues to reconnect with people, her device acquires packets and computes on them to regain access to the vault.

**2) Computation.** User’s device tries different combinations of obtained shares to reconstruct the key. If none of the combinations yields the key, the Apollo application on the device waits for the user to reconnect with the next person. After another reconnection, user’s device

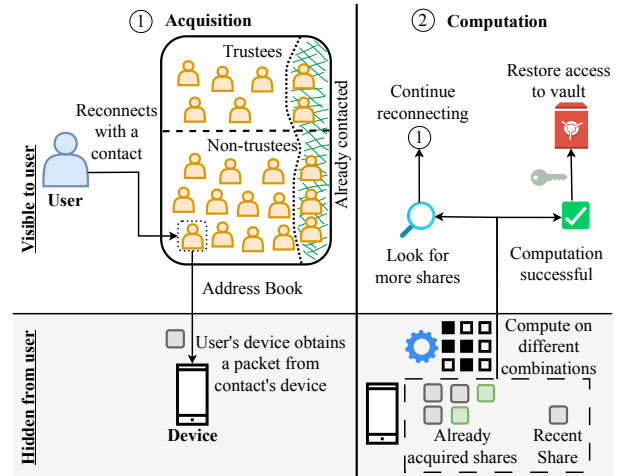


Figure 3. Key recovery.

tries combinations containing shares obtained from the most recent packet. The combinations contain both key as well as random shares, and the key is reconstructed after the user reconnects with *enough* trustees. Thus, Apollo enables key recovery without requiring the user to remember the recovery metadata.

We next provide the details on the multi-layered secret sharing. As a warmup for Apollo’s multi-layered approach, we briefly explain the recovery using a single layer of Shamir’s secret sharing.

## 5.3. Strawman design: Single-layered Approach

This design uses a straightforward approach—it backs up Shamir’s secret shares of the key. Next, we describe the packet structure and the recovery process of this strawman.

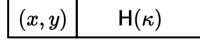


Figure 4. Structure of a blob in the single-layered approach.  $(x, y)$  depicts a secret share and  $H(\kappa)$  is the hash of the key.

**Packet structure.** The single-layered approach backs up one Shamir’s secret share of the key,  $\kappa$ , per trustee, and stores an indistinguishable random share with a non-trustee. The hash of  $\kappa$  ( $h = H(\kappa)$ ) is concatenated with a share to create a blob and the hash acts as indicator of successful reconstruction. Fig. 4 depicts the structure of this blob. The packet,  $\text{pkt}_i$ , held by each person in the address book,  $b_i$ , contains exactly one blob. Thus, assuming the random oracle model, the packets are indistinguishable and they do not leak any extraneous information, making it impossible to determine if the packetholder is a trustee or non-trustee.

**Key Recovery.** After reconnecting with  $n_{\text{obt}}$  people, user’s new device has a set of packets  $P_{\text{obt}} = \{\text{pkt}_1, \dots, \text{pkt}_{n_{\text{obt}}}\}$ . Let  $B_{\text{obt}} = \{b_1, \dots, b_{n_{\text{obt}}}\}$  be the set of people in the address book that user has reconnected with,  $\tau$  be the threshold set by the user, and  $B_T$  be the set of trustees. Then,  $P_{\text{obt}}$  is fed to the reconstruction algorithm:

$$\text{Reconstruct}_{\text{single}}(P_{\text{obt}}) = \begin{cases} \kappa & \text{if } |(B_{\text{obt}} \cap B_T)| \geq \lfloor \tau |B_T| \rfloor \\ \perp & \text{otherwise} \end{cases}$$

We provide the algorithm for Reconstruct in Algorithm 2 due to space constraints. Reconstruct tries all possible combinations of shares in the packets with all possible thresholds. If the output is  $\perp$ , the device waits for the next reconnection. Thus,  $\text{Reconstruct}_{\text{single}}$  reconstructs the key after a user reconnects with a threshold number of trustees. However, this design is not scalable due to the exponential and polynomial growth of the number of combinations with the threshold and the address book size respectively (we provide the bounds on this growth in Appendix C). For instance,  $\text{Reconstruct}_{\text{single}}$  spends 25 hours to compute over all possible combinations after a user reconnects with merely 31 contacts. To make Apollo scale with the threshold and the address book size, we next provide the novel *Multi-layered Secret Sharing* scheme.

## 5.4. Multi-layered Secret Sharing

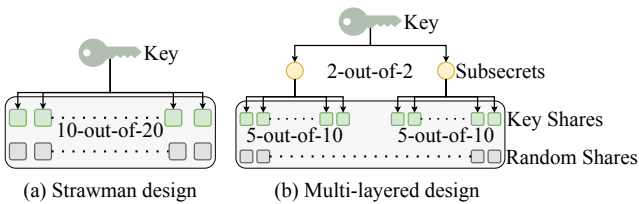


Figure 5. Secret distribution structure.

Multi-layered secret sharing (MLSS) addresses the scalability issue of the single-layered approach. Unlike the single-layered approach, which uses only one layer of shares, MLSS employs two. Instead of backing up Shamir’s secret shares of  $\kappa$ , MLSS generates *subsecrets* of  $\kappa$ , which are additive shares of  $\kappa$ , and distributes the Shamir’s secret shares of these subsecrets among trustees (key shares). Shares of these subsecrets are generated with a fixed threshold, referred to as the *absolute threshold* ( $\alpha$ ). Each subsecret has  $\lfloor \frac{\alpha}{\tau} \rfloor$  shares, ensuring a threshold of at least  $\tau$  for every subsecret. For non-trustees, the random shares mimic Shamir’s secret shares of the subsecrets but are irrelevant for  $\kappa$ ’s recovery. Fig. 5 illustrates the shares generation in MLSS compared to the single-layered approach with 20 trustees,  $\tau = 0.5$  and  $\alpha = 5$ . By using additive shares for subsecrets, we ensure that a user must obtain *at least* the same fraction of shares as the threshold ( $\tau$ ) she set. Therefore, we use additive secret sharing in the subsecrets layer for MLSS. We provide a design using Shamir’s secret sharing in the subsecrets layer in Appendix C. Since one needs to recover all the subsecrets, using multiple layers enables MLSS to use a low threshold for subsecrets while ensuring a high overall threshold for the main secret ( $\kappa$ ).

**Share distribution.** For each subsecret,  $\kappa_l$ , MLSS generates  $\nu_{\text{sub}} = \lfloor \alpha/\tau \rfloor$  shares with a threshold of  $\alpha$ . In total, Apollo generates  $\nu_{\text{total}} = \nu_{\text{sub}} \times \beta$  shares, where  $\beta$  is the number of subsecrets. Apollo randomly samples  $\nu_{\text{min}} = \lfloor \nu_{\text{total}}/|B_T| \rfloor$  shares for each trustee. If some shares are left, i.e.,  $\nu_{\text{total}} \pmod{|B_T|} \neq 0$ , Apollo assigns the remaining shares uniformly among some trustees. This way MLSS attempts to distribute key shares uniformly among trustees. A uniform distribution is more secure as no trustee holds more relevant information than others. We analyze this claim theoretically in Appendix B.6. However, this assignment would make some packets distinguishable, as they would hold more shares than others. Thus, MLSS assigns a random share to all trustees who are assigned  $\nu_{\text{min}}$  shares, while each non-trustee is assigned  $(\nu_{\text{min}} + 1)$  random shares. Hence, every contact holds the same number of distinct shares. When  $\nu_{\text{total}} \pmod{|B_T|} = 0$ , each trustee and non-trustee stores  $\nu_{\text{min}}$  key shares and random shares, respectively. We next explain how tag is stored in a blob to indicate recovery success, while maintaining indistinguishability.

**Number of subsecrets.** Fixing the number of subsecrets across the system would leak information since the number of blobs in a packet would correspond to the threshold. Instead, Apollo fixes the number of shares held by each contact ( $\gamma$ ), which prevents the leakage of information from packet size. This allows Apollo to set a low  $\gamma$ , which improves performance since the non-trustees, as a whole, end up holding fewer random shares. Furthermore, multiple values of the number of subsecrets ( $\beta$ ) can result in the same number of shares per person ( $\gamma$ ). Consider  $\alpha = 3$ ,  $\tau = 0.5$ ,  $|B_T| = 20$ . Consider two possible number of subsecrets:  $\beta_1 = 6, \beta_2 = 5$ . In the latter case, only 10 trustees hold 2 key shares, and the rest hold a key share and a random share. In the former, 16 trustees hold 2 key shares



and 4 of them hold 1. Using  $\beta_1$  is better since: (i) more subsecrets need to be recovered, making malicious recovery harder; (ii) distributing fewer or no random shares among trustees brings MLSS’s recovery probability distribution to resemble that of a single-layered setting. We back this claim empirically and theoretically in Appendix B.4.

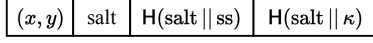


Figure 6. Structure of a blob in MLSS; ss depicts the subsecret to which the share corresponds to.

**Packet structure.** Every contact is assigned one or more shares, and for each share, a blob is created (thus, a packet in MLSS is an array of blobs). For successful recovery indication, MLSS must store some tag in a blob. The structure of the blob has been depicted in Fig. 6. If we had employed the design of the single-layered approach Section 5.3, we would need to store the hash of all the subsecrets and the hash of the key with every contact to ensure indistinguishability. However, this design not only uses more space in contacts’ devices but also makes recovery algorithm slower by making it check with all the subsecrets. To deal with this, MLSS uses cryptographic salts for randomizing the tag. With this approach, a packet stores the salted hash of the relevant subsecrets. This keeps the packet size short and enables the recovery algorithm to check with the relevant hash only. For blobs containing random shares, two (pseudo)random strings are added to ensure indistinguishability. Thus, assuming the random oracle model, this packet structure ensures indistinguishability.

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**Algorithm 1**  $\text{Reconstruct}_{\text{MLSS}}$

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1: Input:  $P_{\text{obt}} = \{\text{pkt}_i\}_{i=1}^{n_{\text{obt}}}, \alpha$ 
2: Output:  $\kappa_{\text{rec}} \in \{\kappa, \perp\}$ 
3:  $K = \{\kappa_i\} \leftarrow \text{ExtractShares}(P_{\text{obt}})$ 
4:  $H \leftarrow \text{GetHashDict}(P_{\text{obt}})$ 
5:  $R \leftarrow \text{GetSaltDict}(P_{\text{obt}})$ 
6:  $\kappa_{\text{rec}} \leftarrow \perp, \text{recovered} \leftarrow \text{false}, S \leftarrow []$ 
7:  $C = \{\{\kappa_i\}_{i=1}^\alpha\} \leftarrow \text{GetCombinations}(K, \alpha)$ 
8: for  $c \in C$  do
9:    $\text{ss}_{\text{int}} \leftarrow \text{Interpolate}(c)$ 
10:   $H_{\text{relevant}} \leftarrow H[c[0]], \text{salt} \leftarrow R[c[0]], \kappa_{\text{obt}} \leftarrow \perp$ 
11:  for  $h \in H_{\text{relevant}}$  do
12:    if  $H(\text{salt} || \text{ss}_{\text{int}}) = h$  and  $\text{ss}_{\text{int}} \notin S$  then
13:       $\text{append}(S, \text{ss}_{\text{int}})$ 
14:       $\kappa_{\text{obt}} = \text{Add}(S)$ 
15:    if  $\kappa_{\text{obt}} \neq \perp$  and  $\text{len}(S) \neq 1$  then
16:      for  $h \in H_{\text{relevant}}$  do
17:        if  $H(\text{salt} || \kappa_{\text{obt}}) = h$  then
18:           $\kappa_{\text{rec}} \leftarrow \kappa_{\text{obt}}, \text{recovered} \leftarrow \text{true}$ 
19:          break
20:    if  $\text{recovered} = \text{true}$  then
21:      break
22: return  $\kappa_{\text{rec}}$ 

```

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**Key Reconstruction.** After acquiring  $n_{\text{obt}}$  packets, the obtained packets,  $P_{\text{obt}} = \{\text{pkt}_1, \text{pkt}_2, \dots, \text{pkt}_{n_{\text{obt}}}\}$ , are fed to the reconstruction algorithm of Apollo,  $\text{Reconstruct}_{\text{MLSS}}$ , which is depicted in Algorithm 1.  $\text{Reconstruct}_{\text{MLSS}}$  generates combinations of  $\alpha$  shares, and tries reconstructing subsecrets from them. If there is a match with one of the hashes in the packet, then the device stores the recovered subsecret. As subsecrets are recovered,  $\text{Reconstruct}_{\text{MLSS}}$  tries to obtain  $\kappa$  from them, and successful reconstruction is indicated by the match of the salted hash.

**Performance gain.** In the single-layered approach, the reconstruction algorithm needs to try all possible thresholds. Hence, for large thresholds, the number of combinations grows rapidly with each reconnection. For example, if a user has already reconnected with 50 people, and if the recovery algorithm is trying a threshold of 10, then an additional reconnection increases the number of combinations by  $2 \times 10^9$ . On the other hand, MLSS ensures only a particular threshold ( $\alpha$ ) is used for generating combinations. Hence, there is only a polynomial growth with every person approached caused by the additional shares obtained. For instance, if we consider the same example as the single-layered approach and consider the absolute threshold,  $\alpha = 3$ , then the number of combinations only grows by  $10^4$ , reducing the number of combination by  $10^5$  times.

## 5.5. Security against malicious recovery

Since no information is leaked by the backed up data, the adversary cannot identify a trustee by simply looking at a packet. The adversary would have to monitor all the aspects of a user’s life to have a reasonable shot at finding trustees. Then, the adversary would compromise the potential trustees. If she fails to recover the key, then she would try extorting from more people, in the hopes of finding a trustee. However, extortion is probabilistic due to the hindrances that an adversary would have to face. For instance, an adversary may fail to deceive a tech-savvy contact with phishing or she may fail to coerce a contact since the contact is situated in a different continent or she may have to pay a large sum of money as a bribe. These uncertainties during recovery, represented by the error in identifying trustees and the failure of extortion, are characterized by the probabilities described in Section 4.3. This is a viable approach for security in a setting where a user loses here device(s) since the user can get shares from contacts for free (by reconnecting), while the adversary has to expend substantial effort for each extortion. Furthermore, the adversary risks getting reported which could impede her recovery attempt, thus further reducing her chances of success. Therefore, Apollo exacerbates the chances of an adversary trying to reconstructing a user’s vault’s key, thereby safeguarding the vault from unauthorized accesses.

## 5.6. Achieving the System Goals

We summarize how Apollo achieves system goals (Section 1.1).

- *Low memorability.* Apollo enables a user to recover her key by reconnecting with her contacts. If a user forgets the recovery metadata, it does not impede recovery. The user can obtain access to the vault if she continues to reconnect as the threshold for recovery is eventually satisfied.
- *Confidentiality.* Since the packets backed up in Apollo are indistinguishable, no recovery metadata is leaked and no one can detect if the packet holds relevant data, which in turn protects trustee privacy Section 3.3.
- *Unauthorized access mitigation.* Due to the inclusion of non-trustees, the trustees are hidden within a larger population and hence, the adversary has to exert more effort as she would obtain chaff packets from non-trustees. This makes malicious recovery a daunting task and we evaluate this gain in security in Section 6.6.
- *Efficiency.* The multi-layered approach generates combinations with a small fixed threshold, which bypasses the exponential growth. We depict the performance gain in Sections 6.4 and 6.5.

## 6. Implementation and Evaluation

In this section, we evaluate Apollo’s performance and empirically analyze its security in terms of *key recovery probability*. In particular, we aim to demonstrate that:

- *C1:* The user can eventually recover her key despite not remembering the exact set of trustees (Section 6.3).
- *C2:* The use of anonymity set only introduces a moderate overhead (Section 6.4).
- *C3:* The cumulative recovery time remains small for large thresholds and large address books (Section 6.5).
- *C4:* The adversary’s probability of reconstructing a key is at least  $100\times$  less than the user (Section 6.6).

### 6.1. Implementation

We implemented Apollo using Go version 1.22 [22], in 7655 lines of code as counted by CLOC [42]. We use SHA256 as hash function. The key,  $\kappa$ , is an array of elements from binary extension field of degree 16.

### 6.2. Experimental Configuration

We ran the experiments on AMD EPYC 7702 Processor, with 16GB RAM and 4 cores.

For presenting the performance gain due to the multi-layered approach Section 5.4 over the single-layered approach Section 5.3, we measured the CPU and wallclock time of the recovery algorithms. The CPU time depicts the resources required for recovery while the wallclock time represents the delay between when a user reconnects with a contact and when the vault recovers. Since the CPU and wallclock time have the same trend, we report the variation of wallclock time in Appendix C.

For depicting a user’s advantage over an adversary in key recovery, we evaluate the key recovery probability based on the probabilities mentioned in Section 4.3. We simulate

a user or an adversary approaching people to obtain packets. The probability is calculated as the ratio of number of successful simulation runs to the total number of runs and we present the cumulative distribution function of this probability. Each probability is calculated over 1 million runs of key recovery simulation.

### 6.3. User key recovery probability

In this experiment, we evaluate the probability of key recovery as a function of a user’s memory error. We use the following parameters for our simulations: (i) percentage recovery threshold of 50% ( $\tau$ ), (ii) an absolute threshold (Section 5.4) of 3 for MLSS, (iii) 20 trustees, which is based on a study for identifying ties between people [43]; (iv) an address book of size 150, which is the Dunbar’s number [44]. To simulate a mistake in recalling, we toss a coin using the memory error as probability.

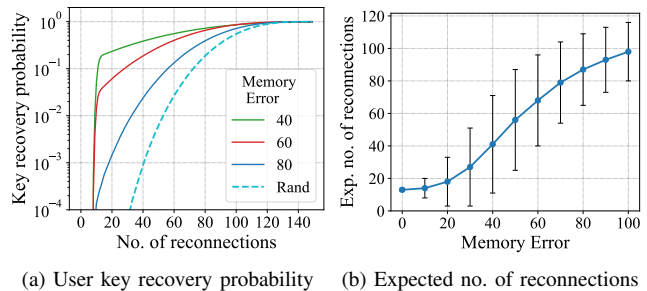


Figure 7. Impact of memory error: (a) variation of key recovery probability with each reconnection; (b) expected number of reconnections for recovery as a function of memory error.

Fig. 7a depicts the probability of key recovery after each reconnection. It shows that the recovery probability eventually becomes 1, thus illustrating that a user can recover her key as long as she can recall and reconnect within her social circle. Therefore, the evaluation claim *C1* holds.

Fig. 7b depicts the expected number of people that need to contact for recovery, as a function of the memory error, with error bars represent the standard deviation. We observe that the error increases initially and then, it decreases since recovery with larger memory errors, such as 80% in Fig. 7a, starts resembling a user reconnecting randomly. Hence, the variance in number of people to be contacted reduces.

### 6.4. Overhead of anonymity set

In this experiment, we evaluate the overhead in terms of the running time of the recovery algorithm with each added reconnection as well as the bandwidth consumed during the backup phase. We use the same parameters as in Section 6.3, except that we evaluate the running time for address books larger than 150 to demonstrate Apollo’s scalability.

Fig. 8 depicts the time taken by the recovery using MLSS as compared to the single layered approach to compute on all combinations after a new reconnection. It shows

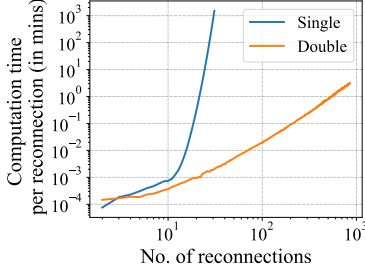


Figure 8. Computation time after every reconnection.

that recovery using MLSS provides an improvement of 5 orders of magnitude at 30 reconnections. However, for the first 3 reconnections, the computation time of the single layered approach is  $0.7 \times -0.9 \times$  of that using MLSS. It is because each contact holds two shares in MLSS as compared to one in the single-layered setting. At the 4th reconnection, we observe that MLSS overtakes the single-layered approach and its computation time is  $1.1 \times$  its single-layered counterpart. It is because the impact of exponential growth in the number of combinations in the single layered approach dominates the impact of multiple shares held by each contact in MLSS. The exponential growth significantly subdues the impact of more shares in MLSS which leads to an improvement of  $740k \times$  at 30 reconnections. The single-layer approach took 25 hours for 30 reconnections, making it impractical to report for more reconnections, while MLSS handled 850 reconnections in just 155 seconds. Thus, the use of anonymity set is made practical in Apollo due to MLSS.

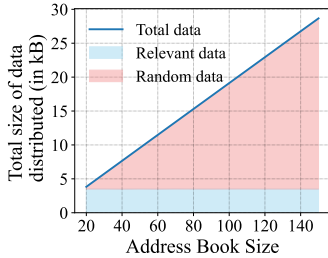


Figure 9. Size of data distributed across the address book.

Fig. 9 depicts the variation of the total bandwidth consumed with the address book size during key backup. It shows that the bandwidth grows linearly with the address book size. The linear growth is due to the increase in random data backed up, as depicted by the red region in Fig. 9. Furthermore, the bandwidth consumed remains under 30kB for 150 contacts. Thus, the computation time and the bandwidth consumed depict that the use of anonymity in Apollo only introduces a moderate overhead and hence, claim *C2* holds.

### 6.5. Scalability of recovery algorithm

In this experiment, we evaluate the recovery algorithm’s scalability based on the expected cumulative recovery time.

The cumulative recovery time is the sum of computing all share combinations until the key is recovered. We consider a user reconnecting in arbitrary order, as it represents the worst-case scenario. We report the cumulative time averaged over 100 runs. We use the same parameters as Section 6.3, however for the single-layered approach and MLSS with absolute threshold  $> 3$ , we ran the experiments with address book of size  $< 100$  since some runs took more than 10 hours which made averaging over 100 runs impractical.

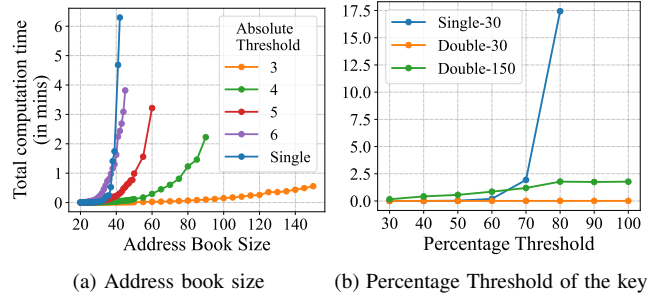


Figure 10. Total computation time: (a) variation with address book size; (b) variation with percentage threshold with 30 contacts for single-layered, and 30 and 150 for double-layered.

Fig. 10a shows the variation in total expected computation time with the address book size and demonstrates the impact of using larger absolute thresholds. It shows that the computation time increases rapidly for absolute thresholds  $> 3$  and hence, we set the absolute threshold to 3 in Apollo. Fig. 10b the variation of total recovery time with the threshold. We observe that in MLSS, the time taken remains almost constant with varying thresholds. In contrast, we observe that the time taken in single-layered approach grows exponentially. Even while using a 5 times larger (150) address book, MLSS takes 7 times less time for recovery for threshold  $\geq 80$ . The improvement in MLSS is due to the use of a small absolute threshold (3) for generating combinations. We conclude Apollo scales with both address book size and threshold and thus, claim *C3* holds.

### 6.6. Security against malicious recovery

In this experiment, we evaluate the adversary’s recovery probability to demonstrate Apollo’s ability to defend against malicious recovery attempts. While trying to acquire shares for maliciously reconstructing a key, the adversary has some uncertainty in obtaining shares, makes a higher error than user in identifying trustees, and runs the risk of getting reported (Section 4.3). We incorporate each of these aspects in our simulations using a coin toss and use the corresponding percentages as the probability for tossing. We use the same parameters as Section 6.3. We assume that a user has 40% memory error, while an adversary has 60%. We set the probability of successful extortion to 25%, consistent with a large-scale phishing study [45]. We set the probability of an adversary getting reported to 5% which is half the reporting probability in [45]. To show the efficacy of reporting, we

include an *impunity* in our plots—an adversary that never gets caught. However, these probabilities do not encompass all kinds of adversaries (e.g., bribing, coercing). Hence, we analyze Apollo’s security against stronger adversaries—an adversary with low error rate in identifying trustees, and another with higher extortion success rate. Validating these percentages would require a user study, which we leave for future work.

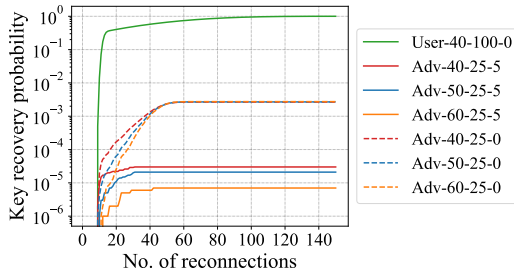


Figure 11. Security of Apollo: Adv-40-25-5 depicts an adversary with 40% error in finding trustees, 25% chance of extortion, and 5% chance of getting reported; dashed curves depict an impunity that never gets caught.

Fig. 11 shows the advantage of the user over a phishing adversary. We observe that the probability of malicious recovery drops below 0.005% which is due to the uncertainty in extorting shares (25%) and the error in identifying trustees (60%) reduce the chances of obtaining enough shares. When we consider an adversary with the same error rate as the user in identifying trustees (40%), the probability remains below 0.01%. Furthermore, the probability of recovery remains below 0.3% for an impunity. Therefore, even for an adversary that makes the same memory error as the user, Apollo can defend even without any reporting.

Fig. 12 presents Apollo’s defense against a strong adversary (e.g., repressive regime), with 50% and 75% chances of extorting shares. We observe that with 5% chance of getting reported, the probability of success for an adversary with 50% extortion rate is less than 0.2%, while an adversary with 75% extortion rate has less than 1.8% chances of success. The defense against such a strong adversary is possible due to reporting and hence, if there is no reporting (impunity), then the chances of success with 75% extortion success rate,

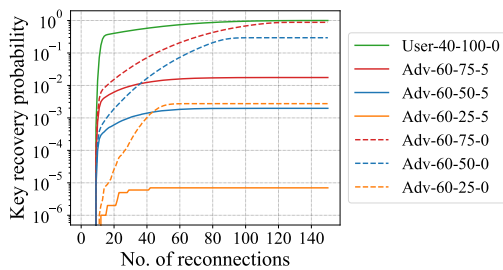


Figure 12. Security against a strong adversary; Adv-40-50-5 shows an adversary with 40% error in finding trustees, 50% chance of extortion, and 5% chance of getting reported; dashed curves depict an impunity that never gets caught.

adversary’s key recovery probability increases to 88%. In summary, for an adversary that has limited ability to extort ( $< 25\%$ ), the anonymity set of Apollo provides a strong defense, while against an adversary that has strong extortion abilities ( $\geq 50$ ), a small reporting probability (5%) plays a key role in protecting user’s key. Hence, claim *C4* holds.

We conclude that a user eventually recovers her vault’s key with Apollo, which reconstructs the key within 3 minutes of reconnecting with a new contact. On the other hand, Apollo protects a user’s key from being maliciously reconstructed by anyone other than the user.

## 7. Discussion

This section presents design aspects related to privacy pool and MLSS that impact the recovery using Apollo.

### 7.1. Probabilistic Recovery

Although MLSS ensures efficient recovery, it might require a user to connect with more trustees than  $\lfloor \tau |B_T| \rfloor$ , due to its two-layered structure. For instance, consider a user approaches 12 trustees in an order such that she obtains shares as shown in Fig. 13. Despite contacting more than  $\tau$  fraction of her trustees, the user would fail to reconstruct  $\kappa$  since one of the subsecrets is not recovered. Although this caveat in MLSS demands more than threshold trustee reconnections from the user, it reduces the probability of reconstruction from an adversary’s perspective too. We provide more details on empirical analysis of the probabilistic recovery in Appendix C and present three variants of Apollo in Appendix C for increasing the probability of recovery at the user specified threshold.

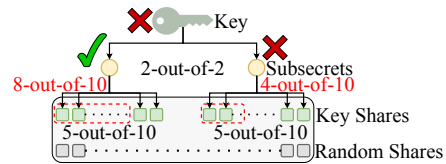


Figure 13. Non-recovery of key despite exceeding threshold since the number of shares for one of the subsecrets is less than the threshold.

### 7.2. Probability ranges for security

We discussed three parameters in Section 4.3 for security: (i)  $k_{\text{err}}$ : the ratio of adversary’s error in identifying trustees to user’s memory error; (ii)  $k_{\text{obt}}$ : the ratio of user’s data obtaining probability to that of the adversary’s extortion probability; and (iii)  $p_{\text{rpt}}$ : the reporting probability. Fig. 11 shows Apollo can defend against an adversary even if the adversary knows the user’s social circle as well as the user does, and hence  $k_{\text{err}} \geq 1$ . It is the latter two parameters which impact the security. Fig. 12 shows that Apollo can provide defense until 75% extortion probability, which leads to  $k_{\text{obt}} \geq \frac{4}{3}$ . With 5% reporting probability, Apollo could

defend against adversaries with extortion probabilities up to 75%. We would need a higher reporting probability ( $\geq 5\%$ ) for defending against stronger adversaries ( $k_{\text{obt}} > \frac{4}{3}$ ).

### 7.3. Practical considerations

**Integration with address book.** We envision Apollo being synchronized with the device’s address book to recommend trustees based on communication data. However, since users often rely on multiple messaging apps for communication [46], default call and messaging records may not accurately reflect communication patterns. An ideal design would use message exchange statistics from multiple applications to produce a more accurate list of trustees.

**Non-responsive contacts.** In Apollo, the backup phase completes once the user’s device receives acknowledgements from *all* devices. If some contacts are offline, this could stall the backup phase. Instead of requiring responses from all contacts, the backup phase could conclude after receiving acknowledgements from a majority of trustees and contacts. Setting a threshold for both the trustees and the contacts is necessary since receiving acknowledgements from *enough* contacts does not imply that enough trustees have acknowledged, and vice versa. If the user goes offline before receiving enough acknowledgements, her device can prompt her to remain online to complete the ongoing backup phase. Finally, while choosing trustees, user does so hoping that they would help her if she loses  $\kappa$ . However, over time, some contacts may lose touch with the user and hence, they might not respond to her during recovery. To counter this, user’s device can send out heartbeat signals periodically, and can notify user to update trustees list if a contact’s device does not respond for an extended period.

**Transient relationships.** Trust among people changes over time: the set of trustees and non-trustees can vary. Apollo must allow the update of privacy pool: adding new contacts, changing contact status (e.g., from a trustee to non-trustee) or changing the threshold. Apollo can monitor newly added contacts and suggest updating the trustees based on recent frequent contacts. In addition, transient trust can lead to a security issue: if a user falls out with a threshold number of trustees, those trustees could collude to reconstruct  $\kappa$  using old shares. This risk persists unless old shares are invalidated. This can be done by changing the vault’s key and Apollo can help by keeping track of contacts whose status change from a trustee to a non-trustee. If the user does not fall out with a threshold number of trustees at once, mechanisms for secure data deletion [47], [48], [49], [50] can ensure that contacts delete their older packets. This would prevent the fallen out trustees from reconstructing the key as they might not hold shares of the same version.

## 8. Related Work

**Secret backup.** CanDID [51] introduces a key recovery mechanism that relies on a secret management committee (SMC). A user can recover her key if she proves successful logins into a threshold number of platforms (e.g.,

Facebook, Twitter). Thus, CanDID introduces a circular problem by requiring one secret to retrieve another, while Apollo gets rid of a circular problem by using distributed trust across a user’s entire social circle. Accesor [52] stores encrypted secrets on a server, while the secret’s encryption key is distributed among guardians, but relies on a ledger like blockchain or a trusted-third party. Similarly, CALYPSO [53] offers decentralized data management but relies on a blockchain. Apollo does not require a distributed ledger for its functioning as using blockchain could be an entry barrier for users. Adei et al. [54] address the same issue of secret recovery even when the user loses devices and credentials. However, users need to monitor for illegitimate attempts to prevent malicious recovery attempts, while Apollo protects the user’s without active involvement of the user. There are solutions back up keys with multiple trusted hardware devices [55], [56], however they are vulnerable to security risks of trusted hardware [57], [58].

**Secret sharing variants.** Password-protected secret sharing (PPSS) [59], [60] uses a password to ensure security even if all trustees are compromised, but it requires a user to remember a password, introducing a circular problem like CanDID. Hierarchical secret sharing schemes [61], [62], [63], [64] employ multiple layers (hierarchy) of shares to achieve specific access structures, whereas MLSS uses multiple layers only for scalability. Proactive secret sharing (PSS) [65], [66], [67] enable a set of parties to update the secret shares. Apollo does not need to utilize PSS since the share update will be handled by the user (instead of the contacts). Beck et al. [68] introduce multi-dealer secret sharing (MDSS), where similar to the idea of having an anonymity set with indistinguishable shares, the user reconstructs the secret by combining relevant and irrelevant information with error-correcting codes. Apollo’s approach of using MLSS is orthogonal to MDSS since it focuses on reducing the number of combinations with multiple layers of shares.

**Social authentication.** Some applications use security questions for account recovery [69], [70], but pose serious usability and security issues [71], [72]. Facebook introduced a mechanism called social authentication [73], where a user must identify known people in a set of photos to regain access. However, this method is vulnerable to attacks using social engineering and artificial intelligence [74], [75], [76]. In contrast, Apollo increases the potential targets for an adversary, making such attacks harder to carry out.

## 9. Conclusion

This paper introduces the notion of recovery metadata in social key recovery, emphasizing the importance of concurrently ensuring self-recovery and protecting recovery metadata privacy. Apollo is the first social key recovery protocol that achieves the illustrated two-fold goal of minimizing the memorability burden and maintaining metadata privacy. Our security analysis and performance evaluation confirm that Apollo ensures vault recovery for users, offers strong protection against malicious recovery attempts, and maintains low latency during recovery.

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## Appendix

### 1. Security of Single Layered Approach

In this section, we analyze the security provided by the single-layered design of Section 5.3. We use the notations provided in Table 2. We consider a user or an adversary approaching people in an arbitrary order, i.e., the contact which is to be approached is chosen in a uniformly random manner without replacement.

TABLE 2. NOTATIONS

Notation	Description
$\mathcal{U}$	User
$\mathcal{A}$	Adversary
$\alpha$	Absolute threshold
$\beta$	No. of subsecrets
$\gamma$	No. of shares held per person ( $\gamma$ is an integer $\geq 2$ )
$\delta$	Fraction of people with $\gamma$ key shares ( $\lfloor \delta n_T \rfloor$ hold $\gamma$ key shares)
$\tau$	Fraction of trustees that should be contacted to recover the key
$n_T$	No. of trustees
$n$	No. of contacts
$k$	No. of people contacted
$n_{T,rec}$	Minimum no. of trustees to be contacted to recover the key
$n_{T,rec}^\gamma$	No. of trustees contacted among $n_{T,rec}$ that hold $\gamma$ key shares

**1.1. Loose bounds on key recovery probability.** We start with loose bounds on the key recovery probability in the single layered setting. Let  $X$  be the random variable depicting the no. of contacts reconnected with for key recovery and let  $t = \lfloor \tau n_T \rfloor$ . On approaching a contact, consider the probability of that contact being a trustee be  $p$ . In other words, we are simulating the reconnection with contacts as a coin toss (where landing a heads corresponds to reconnecting with a trustee). The probability of successfully recovering the key after approaching  $k$  contacts is the probability of contacting  $t$  trustees out of  $k$  contacts. This means we need the probability of obtaining  $t$  heads after  $k$  tosses, which leads to a binomial distribution. Then, we have:

$$P(X \leq k) \approx \binom{k}{t} p^t (1-p)^{k-t} \quad (1)$$

Using the bounds for combinatorial operators (Appendix C), we have:

$$\left(\frac{ek}{t}\right)^t p^t (1-p)^{k-t} < P(X \leq k) < \left(\frac{k}{t}\right)^t p^t (1-p)^{k-t} \quad (2)$$

**1.2. Exact key recovery probability.** Here, we provide bounds on the recovery probability using the single-layered setting. We count the number of successful cases for reporting the probability of key recovery. Then, the probability of recovering the key at the  $k$ -th reconnection is given by:

$C_1$  = No. of ways of contacting  $(t-1)$  trustees after contacting  $(k-1)$  reconnections

$C_2$  = No. of ways of contacting a trustee at the  $k$ -th reconnection

$C_3$  = No. of ways of reconnecting with  $k$  contacts out of  $n$  contacts

$$\begin{aligned} P(X = k) &= \frac{C_1 \times C_2}{C_3} \\ &= t \times \frac{\binom{n_T}{t} \binom{n-n_T}{k-t}}{\binom{n}{k}} \end{aligned} \quad (3)$$

Using the bounds for combinatorial operators (Appendix C), we have:

$$\begin{aligned} \frac{t}{e^k} \left(\frac{f_1}{f_2}\right)^k \left(\frac{1-f_1}{1-f_2}\right)^{k-t} &< P(X = k) < \\ t e^k \left(\frac{f_1}{f_2}\right)^k \left(\frac{1-f_1}{1-f_2}\right)^{k-t} & \quad (4) \\ \text{,where } f_1 = \frac{n_T}{n} \text{ and } f_2 = \frac{t}{k} & \end{aligned}$$

For the security provided by the anonymity set, we would need the probability of  $P(X \leq k)$ . Hence, we can write the probability as:

$$\begin{aligned} P(X \leq k) &= \sum_{i=1}^{i=k} \frac{\binom{n_T}{i} \binom{n-n_T}{i-t}}{\binom{n}{i}} \\ &= \sum_{i=t}^{i=k} \frac{\binom{n_T}{i} \binom{n-n_T}{i-t}}{\binom{n}{i}} \end{aligned} \quad (5)$$

## 2. Security of Multi-Layered Secret Sharing

**2.1. Loose bounds on key recovery probability.** We consider a user or an adversary approaching people in an arbitrary order, i.e., the contact which is to be approached is chosen in a uniformly random manner without replacement. For MLSS, the recovery algorithm needs to recover all the subsecrets to reconstruct the key. Hence, it is not enough to have a condition on contacting more than a threshold number of trustees since the recovery relies on the way shares are distributed. The probability evaluation needs to take into account the number of trustees contacted and, based on the number of trustees contacted, we need to evaluate the probability of key recovery. Hence, the probability of key recovery can be segregated into two parts:

$P_1$  = Probability of contacting  $k_T$  trustees after contacting  $k$  contacts

$P_2$  = Probability of recovering the key after contacting  $k_T$  trustees

Then, the probability of key recovery would be:

$$P(X = k) = \sum_{k_T} P_1 \times P_2$$

$P_1$ 's value can be obtained from Eq. (1):

$$P_1 \approx \binom{k}{k_T} p^{k_T} (1-p)^{k-k_T} \quad (6)$$

For obtaining an approximate value of  $P_2$ , we need  $\alpha$  shares from all the subsecrets. Therefore, the approximate value of  $P_2$  is:

$$P_2 \approx \frac{\left(\lfloor \frac{\alpha}{\gamma} \rfloor\right)^\beta \times \binom{n_T \gamma - \alpha \beta}{k_T \gamma - \alpha \beta}}{\binom{n_T \gamma}{k_T \gamma}} \quad (7)$$



**2.2. Bounds on key recovery probability.** We use the same idea of splitting the probability into two parts: (i) probability of contacting  $k_T$  trustees after contacting  $k$  contacts; (ii) probability of recovering the key after contacting  $k_T$  trustees.

The exact value of  $P_1$  can be obtained as:

$$P_1 = k_T \times \frac{\binom{n_T}{k_T} \binom{n-n_T}{k-k_T}}{\binom{n}{k}} \quad (8)$$

For evaluating  $P_2$ , we need the following parameters:

$n\gamma$  = Total no. of shares available

$\beta \lfloor \frac{\alpha}{\tau} \rfloor$  = Total no. of relevant shares (shares that can be used to reconstruct the key)

$n_T\gamma - \beta \lfloor \frac{\alpha}{\tau} \rfloor$  = No. of random shares held by trustees

Let  $x_i, i \in \{1, \dots, \beta\}$  depict the no. of shares obtained for subsecret  $i$  and let  $x_r$  be the no. of random shares obtained from trustees.

For evaluating the probability, we use stars-and-bars theory for counting. The total number of ways of obtaining shares after contacting  $k_T$  trustees is given by:

$$\begin{aligned} x_1 + x_2 + \dots + x_\beta + x_r &= k_T\gamma, \text{ where} & (9) \\ 0 \leq x_i &\leq \lfloor \alpha\tau \rfloor, \forall 1 \leq i \leq \beta, \text{ and} \\ 0 \leq x_r &\leq n_T\gamma - \beta \lfloor \frac{\alpha}{\tau} \rfloor \end{aligned}$$

Let the no. of cases from this stars and bars problem be  $C_1$ .

Now, we need to evaluate the no. of successful cases. For the successful cases, we need to have at least  $\alpha$  no. of shares from each subsecret. Hence, the number of successful cases can be counted by:

$$\begin{aligned} x_1 + x_2 + \dots + x_\beta + x_r &= k_T\gamma, \text{ where} \\ \alpha \leq x_i &\leq \lfloor \alpha\tau \rfloor, \forall 1 \leq i \leq \beta, \text{ and} \\ 0 \leq x_r &\leq n_T\gamma - \beta \lfloor \frac{\alpha}{\tau} \rfloor \\ \implies x'_1 + x'_2 + \dots + x'_\beta + x'_r &= k_T\gamma - \alpha\beta, \text{ where} \\ 0 \leq x'_i &\leq \lfloor \alpha\tau \rfloor - \alpha, \forall 1 \leq i \leq \beta, \text{ and} \\ 0 \leq x'_r &\leq n_T\gamma - \beta \lfloor \frac{\alpha}{\tau} \rfloor \end{aligned} \quad (10)$$

For the generic equation,

$$a_1 + a_2 + \dots + a_n = N, \text{ where } 0 \leq a_i \leq r_i$$

The no. of cases of is given by:

$$\sum_{S \subseteq \{1, 2, \dots, n\}} (-1)^{|S|} \binom{N + n - 1 - \sum_{i \in S} (r_i + 1)}{n - 1} \quad (11)$$

Next, we analyze the security of the multi-layered setting with respect to the threshold set by the user ( $\tau$ ). This section starts with the discussion on the choice of using the floor operator ( $\lfloor \cdot \rfloor$ ) while generating shares. Recall that the floor operator applied to a real number returns the largest integer that is at most the real number, e.g.  $\lfloor 5.2 \rfloor = 5$ ,  $\lfloor 5.9 \rfloor = 5$ . Then, this section proceeds to an analysis of threshold

TABLE 3. DEFAULT VALUES

Parameter	Default Value
No. of trustees	20
Anonymity Set Size	150
Percentage threshold ( $\tau$ )	0.5
Absolute Threshold ( $\alpha$ )	3
No. of shares per person ( $\gamma$ )	2
No. of subsecrets ( $\beta$ )	6

provided by the multi-layered approach. Recall that  $\tau$  is the fraction of trustees that  $\mathcal{U}$  needs to contact for recovering  $\kappa$ . We define *perfect security* of multi-layered paradigm as the property where there is no scenario where  $\kappa$  can be recovered after obtaining packets from less than  $\lfloor \tau n_T \rfloor$ . We show that perfect security can be achieved only with specific design choices and then, we show that the probability of violating this security remains very low. This section ends with the analysis of the expected security of MLSS.

**2.3. Choosing the number of shares per subsecret.** We assign  $\lfloor \frac{\alpha}{\tau} \rfloor$  shares to each subsecret. Therefore, we start with describing why we used  $\lfloor \cdot \rfloor$ , instead of  $\lceil \cdot \rceil$  and  $\lceil \cdot \rceil$ , for generating shares for subsecrets. The following observation is useful:

$$\left\lfloor \frac{\alpha}{\tau} \right\rfloor \leq \frac{\alpha}{\tau} \implies \left\lceil \frac{\alpha}{\tau} \right\rceil \geq \tau. \quad (12)$$

Eq. (12) implies that choosing  $\lfloor \cdot \rfloor$  guarantees that the threshold of each subsecret is at least  $\tau$ . Since we use additive secret sharing in MLSS, this guarantees that one needs to obtain at least  $\tau$  fraction of total shares to reconstruct the key. This guarantee would not have been provided if we used  $\lceil \cdot \rceil$  or  $\lceil \cdot \rceil$ . If everyone held one share, then this design would have provided perfect security. Since this is not the case in MLSS, we further analyze the implications of share distribution in MLSS.

**2.4. Perfect Security Guarantees.** Now, we discuss the security guarantees provided by MLSS, and the design choices that should be made in order to achieve perfect security. Given the distribution in MLSS, a trustee either receives  $\gamma$  or  $(\gamma - 1)$  key shares. Therefore, we have

$$\gamma \lfloor \delta n_T \rfloor + (\gamma - 1)(n_T - \lfloor \delta n_T \rfloor) = \left\lfloor \frac{\alpha}{\tau} \right\rfloor \beta, \quad (13)$$

which gives us an expression for the number of subsecrets  $\beta$ , as

$$\beta = \frac{\lfloor n_T(\delta + \gamma - 1) \rfloor}{\left\lfloor \frac{\alpha}{\tau} \right\rfloor}. \quad (14)$$

Eq. (14) depicts that there are only certain values of  $\delta$  that are viable to be used (since  $\beta$  must be an integer). We use this value for our further analysis.

The perfect security of MLSS is defined corresponding to the minimum number of trustees that can be contacted ( $n_{T, \text{rec}}$ ) to recover the key across *all* the distributions of

shares among the trustees and *all* patterns of approach. That is, among all possible permutations of share distribution, and patterns of approach of contacts,  $n_{T,\text{rec}}$  is attained for only some specific distributions of shares, and some patterns of approach. While analyzing the perfect security, we aim to find the existence of settings where a key can be recovered after contacting a less than threshold number of trustees. Therefore, there are distributions that are less secure with respect to the others. MLSS would achieve perfect security if  $n_{T,\text{rec}} \geq \tau n_T$ .

For recovering the key  $\kappa$ ,  $\mathcal{U}$  needs to at least obtain  $\alpha$  number of shares of each subsecret, i.e., the total number of shares obtained is

$$\gamma n_{T,\text{rec}}^\gamma + (\gamma - 1)(n_{T,\text{rec}} - n_{T,\text{rec}}^\gamma) \geq \alpha\beta, \quad (15)$$

where we denote the number of contacted trustees that hold  $\gamma$  shares by  $n_{T,\text{rec}}^\gamma$ .

The minimum value of  $n_{T,\text{rec}}$  can be achieved by obtaining shares from people with  $\gamma$  shares (approaching people with more shares implies that overall less people need to be contacted). Thus, we have two branches from Eq. (15): (i)  $\gamma[\delta n_T] \geq \alpha\beta$ ; and (ii)  $\gamma[\delta n_T] < \alpha\beta$ . In the first case,  $\kappa$  can be recovered by simply approaching people with  $\gamma$  key shares while in the second case both people with  $\gamma$  and  $(\gamma - 1)$  shares need to be approached. Specifically, in the second case, all the people with  $\gamma$  shares will be contacted such that  $n_{T,\text{rec}}$  is minimized. We provide the analyses of security in both the cases below.

Case 1,  $\gamma[\delta n_T] \geq \alpha\beta$ :. In this case,  $n_{T,\text{rec}} = n_{T,\text{rec}}^\gamma \leq [\delta n_T]$ . Therefore, from Eq. (15), we have  $\gamma n_{T,\text{rec}} \geq \alpha\beta$ . Using Eq. (14) and Eq. (12) respectively, we have

$$\begin{aligned} \gamma n_{T,\text{rec}} &\geq \left\lceil \frac{\alpha}{\tau} \right\rceil [n_T(\delta + \gamma - 1)] \\ &\geq \tau [n_T(\delta + \gamma - 1)]. \end{aligned}$$

Dividing both sides by  $\gamma$  and using the relation  $\lfloor x \rfloor > x - 1$ , this reduces to

$$\begin{aligned} n_{T,\text{rec}} &\geq \frac{\tau}{\gamma} [n_T(\delta + \gamma - 1)] \\ &> \frac{\tau}{\gamma} (n_T(\delta + \gamma - 1) - 1) \\ &= \tau \left( n_T - \frac{n_T(1 - \delta) + 1}{\gamma} \right). \end{aligned} \quad (16)$$

On the other hand, since  $n_{T,\text{rec}}$  is the *minimum* number of trustees that need to be contacted, we also have  $\gamma(n_{T,\text{rec}} - 1) \leq \alpha\beta$ . Using Eq. (14) and the relation  $x \geq \lfloor x \rfloor > x - 1$ , this can be manipulated to obtain

$$\begin{aligned} \gamma(n_{T,\text{rec}} - 1) &\leq \left\lceil \frac{\alpha}{\tau} \right\rceil [n_T(\delta + \gamma - 1)] \\ &< \frac{\alpha}{\tau - 1} n_T(\delta + \gamma - 1). \end{aligned}$$

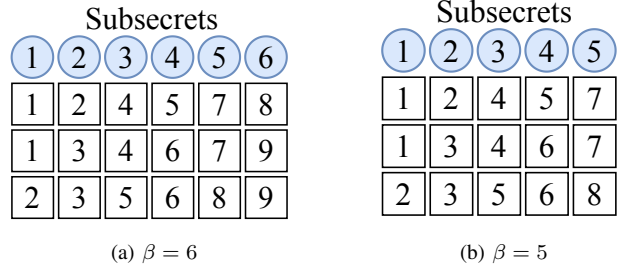


Figure 14. Configuration of shares obtained to reconstruct the key with less than  $\tau n_T$  people

Dividing both sides by  $\gamma$  and rearranging terms, we have

$$\begin{aligned} n_{T,\text{rec}} - 1 &< \frac{\tau}{1 - \frac{\tau}{\alpha}} \frac{n_T(\delta + \gamma - 1)}{\gamma} \\ \iff n_{T,\text{rec}} &< \frac{\tau}{1 - \frac{\tau}{\alpha}} \left( n_T - \frac{n_T(1 - \delta)}{\gamma} \right) + 1. \end{aligned} \quad (17)$$

Combining Eq. (16) and Eq. (17), we have the following upper and lower bounds on  $n_{T,\text{rec}}$ :

$$\tau n_T \left( 1 - \frac{1 - \delta}{\gamma} \right) - \frac{\tau}{\gamma} < n_{T,\text{rec}} < \frac{\tau n_T}{1 - \frac{\tau}{\alpha}} \left( 1 - \frac{1 - \delta}{\gamma} \right) + 1. \quad (18)$$

We now use these bounds to construct examples where perfect secrecy cannot be achieved. Consider the values used in the simulations (Table 3), where  $n_T = 20$ ,  $\alpha = 3$ ,  $\beta = 6$ ,  $\gamma = 2$ , and  $\tau = 0.5$ , then the above bounds are

$$\frac{35}{4} < n_{T,\text{rec}} < \frac{59}{5}. \quad (19)$$

We see that this case may not guarantee perfect security because  $n_{T,\text{rec}}$  can take values lesser than  $\tau n_T = 10$ . In particular, it must be either 9 or 10. We show one possible configuration of recovering  $\kappa$  after contacting 9 people in Fig. 14a. The circles depict the subsecrets and a tile vertically below the subsecret depicts a share of the subsecret. The numbers in the tiles depict serial number of the trustee from whom the share was obtained. Since the absolute threshold is 3, the figures depict 3 shares, which are needed to recover that subsecret.

On the other hand, consider the values of Table 3, except with 5 subsecrets, i.e.,  $\beta = 5$ . Then, we have

$$\frac{29}{4} < n_{T,\text{rec}} < 10. \quad (20)$$

We can immediately see that this case *cannot* guarantee perfect security because  $n_{T,\text{rec}}$  must be lesser than  $\tau n_T = 10$ . As in the previous example, we show one possible configuration of recovering  $\kappa$  after contacting 8 people in Fig. 14b.

Precisely, as depicted by Eq. (18), the lower bound in Eq. (18) is always less than  $\tau n_T$ , but it is closer to the required value if the value of  $\frac{1 - \delta}{\gamma}$  is low. This implies that MLSS is furthest from achieving perfect security if  $\delta \ll 1$ , and closest if  $\delta$  is close to 1. Hence, if the share distribution

ensures that all the trustees have an equal number of key shares  $\gamma$  (in other words, the total number of shares should be a multiple of the number of trustees), then the system can achieve perfect security. For instance consider the following values:  $n_T = 20$ ,  $\alpha = 4$ ,  $\beta = 5$ ,  $\gamma = 2$ , and  $\tau = 0.5$  — the number of key shares in this case is 40 which divides  $n_T$ . If we evaluate the bounds in Eq. (18) for this case, we obtain

$$\frac{39}{4} < n_{T,\text{rec}} < \frac{87}{7}. \quad (21)$$

Eq. (21) implies that for all the configurations the value of  $n_{T,\text{rec}}$  is at least 10, which is  $\tau n_T$ . Therefore, making the total number of key shares a multiple of the number of trustees provides the system perfect security. For ensuring this requirement, MLSS might need to increase  $\beta$  (and hence,  $\gamma$ ) so that the total number of shares is a multiple of  $n_T$ .

Furthermore, if  $\gamma \rightarrow \infty$ , then  $\frac{n_T(1-\delta)+1}{\gamma} \rightarrow 0$ . Thus, the larger the value of  $\gamma$ , the more ideal the design. Conversely, it implies that Apollo would require to compute on more number of shares (non-trustees would also hold more random shares, resulting in more random shares in the system), thus making the solution inefficient. Instead, MLSS tries to set  $\delta$  as close to 1 as possible for a smaller  $\gamma$  such that the design is secure, while being efficient at the same time. This feature not only improves the security but also reduces the overall computation time.

Case 2,  $\gamma \lfloor \delta n_T \rfloor < \alpha \beta$ : In this case, denote the number of trustees contacted among  $n_{T,\text{rec}}$  that hold  $\gamma$  key shares by  $n_{T,\text{rec}}^\gamma = \lfloor \delta n_T \rfloor$ . Therefore, from Eq. (15), we have

$$\gamma \lfloor \delta n_T \rfloor + (\gamma - 1)(n_{T,\text{rec}} - \lfloor \delta n_T \rfloor) \geq \alpha \beta.$$

By simplifying the left-hand side and using Eq. (14) and Eq. (12), we have

$$\begin{aligned} (\gamma - 1)n_{T,\text{rec}} + \lfloor \delta n_T \rfloor &\geq \alpha \beta \\ &= \alpha \frac{\lfloor n_T(\delta + \gamma - 1) \rfloor}{\lfloor \frac{\alpha}{\tau} \rfloor} \\ &\geq \alpha \frac{\lfloor n_T(\delta + \gamma - 1) \rfloor}{\frac{\alpha}{\tau}} \\ &= \tau \lfloor n_T(\delta + \gamma - 1) \rfloor. \end{aligned} \quad (22)$$

Subtracting  $\lfloor \delta n_T \rfloor$  from both sides and using the relation  $x - 1 < \lfloor x \rfloor \leq x$ , this reduces to

$$\begin{aligned} (\gamma - 1)n_{T,\text{rec}} &\geq \tau \lfloor n_T(\delta + \gamma - 1) \rfloor - \lfloor \delta n_T \rfloor \\ &> \tau(n_T(\delta + \gamma - 1) - 1) - \delta n_T. \end{aligned} \quad (23)$$

Dividing both sides by  $\gamma - 1$ , we obtain

$$\begin{aligned} n_{T,\text{rec}} &> \frac{\tau}{\gamma - 1}(n_T(\delta + \gamma - 1) - 1) - \frac{\delta n_T}{\gamma - 1} \\ &= \tau n_T + \frac{\delta \tau n_T}{\gamma - 1} - \frac{\tau}{\gamma - 1} - \frac{\delta n_T}{\gamma - 1} \\ &= \tau n_T - \frac{\delta n_T(1 - \tau)}{\gamma - 1} - \frac{\tau}{\gamma - 1}, \end{aligned} \quad (24)$$

i.e., a lower bound to  $n_{T,\text{rec}}$ .

On the other hand, since  $n_{T,\text{rec}}$  represents the minimum number of people that need to be contacted across all possible distributions, the key should not be recovered after contacting  $(n_{T,\text{rec}} - 1)$  people. This gives us

$$\gamma \lfloor \delta n_T \rfloor + (\gamma - 1)(n_{T,\text{rec}} - 1 - \lfloor \delta n_T \rfloor) \leq \alpha \beta,$$

which can be rearranged as before to obtain

$$(\gamma - 1)(n_{T,\text{rec}} - 1) + \lfloor \delta n_T \rfloor \leq \alpha \beta.$$

Using Eq. (14) to replace  $\beta$ , and the relation  $x - 1 < \lfloor x \rfloor \leq x$ , we have

$$\begin{aligned} (\gamma - 1)(n_{T,\text{rec}} - 1) + \lfloor \delta n_T \rfloor &\leq \alpha \frac{\lfloor n_T(\delta + \gamma - 1) \rfloor}{\lfloor \frac{\alpha}{\tau} \rfloor} \\ &< \alpha \frac{\lfloor n_T(\delta + \gamma - 1) \rfloor}{\frac{\alpha}{\tau} - 1} \\ &= \tau \frac{\lfloor n_T(\delta + \gamma - 1) \rfloor}{1 - \frac{\tau}{\alpha}}. \end{aligned}$$

Subtracting  $\lfloor \delta n_T \rfloor$  from both sides and using the relation  $x - 1 < \lfloor x \rfloor \leq x$ , we obtain

$$\begin{aligned} (\gamma - 1)(n_{T,\text{rec}} - 1) &\leq \tau \frac{\lfloor n_T(\delta + \gamma - 1) \rfloor}{1 - \frac{\tau}{\alpha}} - \lfloor \delta n_T \rfloor \\ &< \tau \frac{n_T(\delta + \gamma - 1)}{1 - \frac{\tau}{\alpha}} - (\delta n_T - 1). \end{aligned}$$

Dividing both sides by  $\gamma - 1$ , we have

$$n_{T,\text{rec}} - 1 < \frac{\tau}{\gamma - 1} \frac{n_T(\delta + \gamma - 1)}{1 - \frac{\tau}{\alpha}} - \frac{\delta n_T - 1}{\gamma - 1}.$$

Rearranging terms, the above inequality can be written as

$$\begin{aligned} n_{T,\text{rec}} &< \frac{\tau n_T}{1 - \frac{\tau}{\alpha}} - \frac{\delta n_T \left(1 - \frac{\tau}{1 - \frac{\tau}{\alpha}}\right)}{\gamma - 1} + \frac{1}{\gamma - 1} + 1 \\ &= \frac{\tau n_T}{1 - \frac{\tau}{\alpha}} - \frac{\delta n_T \left(1 - \frac{\tau}{1 - \frac{\tau}{\alpha}}\right)}{\gamma - 1} + \frac{\gamma}{\gamma - 1}. \end{aligned} \quad (25)$$

Putting Eq. (24) and Eq. (25) together, we obtain the following lower and upper bound to  $n_{T,\text{rec}}$ :

$$\begin{aligned} \tau n_T - \frac{\delta n_T(1 - \tau)}{\gamma - 1} - \frac{\tau}{\gamma - 1} &< n_{T,\text{rec}} < \\ \frac{\tau n_T}{1 - \frac{\tau}{\alpha}} - \frac{\delta n_T \left(1 - \frac{\tau}{1 - \frac{\tau}{\alpha}}\right)}{\gamma - 1} + \frac{\gamma}{\gamma - 1}. \end{aligned} \quad (26)$$

As in the previous case, we use these bounds to show examples where perfect security is not guaranteed. Consider the values used in the simulations (Table 3), where  $n_T = 20$ ,  $\alpha = 3$ ,  $\beta = 4$ ,  $\gamma = 2$ , and  $\tau = 0.5$ , then the above bounds are

$$\frac{15}{2} < n_{T,\text{rec}} < \frac{62}{5}. \quad (27)$$

We see that Eq. (27) does not guarantee perfect security again as  $n_{T,\text{rec}}$  could be 8, 9, or 10, and Fig. 15 shows

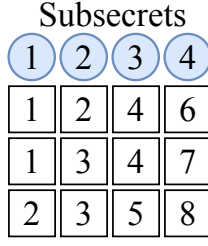


Figure 15. Configuration of shares obtained to reconstruct the key with less than  $\tau n_T$  people with  $\gamma n_T$

one pattern of obtaining shares such that  $\kappa$  is reconstructed after contacting less than  $\tau n_T = 10$  people. As depicted by Eq. (26),  $n_{T,\text{rec}}$  can take values less than  $\tau n_T$ . One way of bringing the system closer to  $\tau n_T$  is by setting  $\delta$  to 0, which again implies a uniform distribution. Another way of ensuring that this does not happen is by increasing the lower limit by increasing the value of  $\gamma$ . That is, similar to Eq. (18), if  $\gamma \rightarrow \infty$ , then  $\frac{\tau}{\gamma-1} + \frac{\delta n_T(1-\tau)}{\gamma-1} \rightarrow 0$ , and thus, the lower limit of  $n_{T,\text{rec}}$  will tend to  $\tau n_T$ . Furthermore, in Eq. (26), with the increase in the value of  $\gamma$ , the lower limit also gets closer to a single layer distribution. However, the term  $\frac{1}{1-\frac{\tau}{\alpha}}$  also impacts the upper bound, and might take the upper bound to a value much higher than  $\tau n_T$ . Although the upper bound does not violate the perfect security, it implies that there are distributions where the value of  $n_T$  could deviate significantly from  $\tau n_T$ , thus demanding more effort from the user. To get the upper limit closer to  $\tau n_T$ , the system could use a higher value of  $\alpha$ . Thus, by setting a high value of  $\gamma$  and  $\alpha$  one can bring all share distributions close to a single-layered setting. However, increasing  $\gamma$  and  $\alpha$  is detrimental to system's performance as the former would lead to increase in the number of shares over which combinations have to be generated, while the latter would lead to more combinations since the reconstruction involves generation of  $\binom{\#\text{shares}}{\alpha}$  combinations.

**2.5. Techniques to achieve perfect security.** When the number of shares is not a multiple of the number of trustees, Apollo can achieve perfect security using some tweaks in MLSS:

1) *Increasing the number of shares per subsecret while keeping the number of subsecrets constant:* Recall that the number of subsecrets was the maximum possible number of subsecrets such that the number of shares distributed among trustees remains constant across the system ( $\gamma$ ). With this design, if we increase the number of shares one-by-one for each subsecret uniformly, then there will be structure of the multi-layered setting where the number of shares will be a multiple of the number of trustees (where some subsecrets will have  $(\lfloor \frac{\alpha}{\tau} \rfloor + 1)$  shares and some of them will have  $\lfloor \frac{\alpha}{\tau} \rfloor$  shares. This design reduces the overall security because some subsecrets have their threshold less than the value set by the user.

2) *Increasing the number of subsecrets and distributing less shares:* The second idea is to enhance the security of the system, while requiring more effort from users' end. In this approach, we increase the number of subsecrets to  $\beta + 1$  and in turn, the number of shares per person becomes  $\gamma + 1$ . Now, if we remove the extraneous shares one-by-one from subsecrets uniformly, then there will be a structure such that the total number of shares will be a multiple of the number of trustees. This design is more secure because some subsecrets will have a higher percentage threshold as compared to others (because some subsecrets will have  $(\lfloor \frac{\alpha}{\tau} \rfloor - 1)$  shares.)

**2.6. Uniform Share Distribution.** Eqs. (18) and (26) provide a reasoning behind using a uniform distribution of shares among trustees — if we ensure that each trustee receives the same number of shares, then the design is equivalent to the single-layered approach. However, these equations do not prove why we chose to start with a uniform distribution, i.e., trustees have either  $(\gamma - 1)$  or  $\gamma$  shares. Therefore, now we describe a more skewed version of share distribution, such that a trustee can hold  $i$  key shares, where  $i \in \{1, 2, \dots, \gamma\}$ . Let  $\delta_i$  be the fraction of trustees that hold  $i$  key shares, i.e.,  $\lfloor \delta_i n_T \rfloor$  hold  $i$  key shares. This is a more general treatment of the previous case, with  $\delta_\gamma = \delta$ ,  $\delta_{\gamma-1} = 1 - \delta$ , and  $\delta_i = 0$  for  $i = 1, \dots, \gamma - 2$ . In this case, we have

$$\sum_{i=1}^{\gamma} \lfloor \delta_i n_T \rfloor i = \left\lfloor \frac{\alpha}{\tau} \right\rfloor \beta, \quad \text{and} \quad (28)$$

$$\sum_{i=1}^{\gamma} \lfloor \delta_i n_T \rfloor = n_T. \quad (29)$$

For minimizing the number of trustees to be contacted for recovering the key ( $n_{T,\text{rec}}$ ),  $\mathcal{U}$  should contact people with more shares first. Therefore, let  $\mathcal{U}$  obtain key shares from all the trustees with  $\{\gamma, \gamma - 1, \dots, i + 1\}$  key shares, and obtain rest of the key shares from people with  $i$  key shares for some number  $i < \gamma$ . For the key to be recovered, we require

$$i \left( n_{T,\text{rec}} - \sum_{j=i+1}^{\gamma} \lfloor \delta_j n_T \rfloor \right) + \sum_{j=i+1}^{\gamma} j \lfloor \delta_j n_T \rfloor \geq \alpha \beta,$$

which can be simplified to obtain

$$i n_{T,\text{rec}} + \sum_{j=i+1}^{\gamma} (j - i) \lfloor \delta_j n_T \rfloor \geq \alpha \beta$$

$$\implies i n_{T,\text{rec}} \geq \alpha \beta - \sum_{j=i+1}^{\gamma} (j - i) \lfloor \delta_j n_T \rfloor.$$

Using Eq. (28) and the relation  $\lfloor x \rfloor \leq x$ , we have

$$\begin{aligned} in_{T,\text{rec}} &\geq \alpha \frac{\sum_{j=1}^{\gamma} \lfloor \delta_j n_T \rfloor j}{\lfloor \frac{\alpha}{\tau} \rfloor} - \sum_{j=i+1}^{\gamma} \lfloor \delta_j n_T \rfloor (j-i) \\ &\geq \alpha \frac{\sum_{j=1}^{\gamma} \lfloor \delta_j n_T \rfloor j}{\frac{\alpha}{\tau}} - \sum_{j=i+1}^{\gamma} \lfloor \delta_j n_T \rfloor (j-i) \\ &= \tau \sum_{j=1}^{\gamma} \lfloor \delta_j n_T \rfloor j - \sum_{j=i+1}^{\gamma} \lfloor \delta_j n_T \rfloor (j-i). \end{aligned}$$

Dividing both sides by  $i$  and breaking the first summation over the indices  $\{1, \dots, i-1\}$ ,  $\{i\}$ , and  $\{i+1, \dots, \gamma\}$ , we have

$$\begin{aligned} n_{T,\text{rec}} &\geq \tau \sum_{j=1}^{\gamma} \lfloor \delta_j n_T \rfloor \frac{j}{i} - \sum_{j=i+1}^{\gamma} \lfloor \delta_j n_T \rfloor \frac{j-i}{i} \\ &= \tau \sum_{j=1}^{i-1} \lfloor \delta_j n_T \rfloor \frac{j}{i} + \tau \lfloor \delta_i n_T \rfloor - \sum_{j=i+1}^{\gamma} \lfloor \delta_j n_T \rfloor \frac{j-\tau j-i}{i} \\ &= \tau \sum_{j=1}^{i-1} \lfloor \delta_j n_T \rfloor \frac{j}{i} + \sum_{j=i+1}^{\gamma} \lfloor \delta_j n_T \rfloor \left( \frac{(\tau-1)j}{i} + 1 \right) \\ &\quad + \tau \lfloor \delta_i n_T \rfloor. \end{aligned} \quad (30)$$

Introducing the notation  $\tau_j = \frac{j}{i}$ , we can write Eq. (30) as

$$\begin{aligned} n_{T,\text{rec}} &\geq \tau \sum_{j=1}^{i-1} \lfloor \delta_j n_T \rfloor \tau_j + \sum_{j=i+1}^{\gamma} \lfloor \delta_j n_T \rfloor ((\tau-1)\tau_j + 1) \\ &\quad + \tau \lfloor \delta_i n_T \rfloor. \end{aligned} \quad (31)$$

Observe that this lower bound is an inequality only because we upper bound  $\lfloor \frac{\alpha}{\tau} \rfloor$  by  $\frac{\alpha}{\tau}$ ; the other steps are all identities. Hence, the tightness of this bound is determined by how close  $\frac{\alpha}{\tau}$  is to an integer relative to itself. Recall that  $\alpha$  is an absolute threshold (which is an integer) and  $\tau$  is the fraction of trustees to be contacted to recover the key (which is between 0 and 1). This implies that  $\frac{\alpha}{\tau}$  is guaranteed to be a large number, and hence the relative error in approximating  $\lfloor \frac{\alpha}{\tau} \rfloor$  by  $\frac{\alpha}{\tau}$  is small. In particular, for the values in Table 3, we have  $\frac{\alpha}{\tau} = 6$ , which is already an integer. Thus, we conclude that this lower bound can be taken as a heuristic approximation.

On the other hand, multiplying both sides of Eq. (28) by  $\tau$  and rearranging the left-hand side, we obtain

$$\tau \sum_{j=1}^{i-1} \lfloor \delta_j n_T \rfloor + \sum_{j=i+1}^{\gamma} \tau \lfloor \delta_j n_T \rfloor + \tau \lfloor \delta_i n_T \rfloor = \tau n_T. \quad (32)$$

Now, observe that for  $j < i$ , since  $\tau_j = \frac{j}{i}$ , we have  $\tau \lfloor \delta_j n_T \rfloor \tau_j < \tau \lfloor \delta_j n_T \rfloor$ , and hence

$$\sum_{j=1}^{i-1} \tau \lfloor \delta_j n_T \rfloor \tau_j < \sum_{j=1}^{i-1} \tau \lfloor \delta_j n_T \rfloor. \quad (33)$$

Meanwhile, for  $j \in \{i+1, \dots, \gamma\}$ , since  $\tau_j > 1$ , we have:

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### Algorithm 2 Reconstruct<sub>single</sub>

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1: Input:  $P_{\text{obt}} = \{\text{pkt}_i\}_{i=1}^{i=n_{\text{obt}}}$ 
2: Output:  $\kappa_{\text{rec}} \in \{\kappa, \perp\}$ 
3:  $E = \{s_i\}_{i=1}^{i=n_{\text{obt}}} \leftarrow \text{ExtractShares}(P_{\text{obt}})$ 
4:  $h \leftarrow \text{ExtractHash}(P_{\text{obt}})$ 
5:  $t \leftarrow 2, \kappa_{\text{rec}} \leftarrow \perp, \text{recovered} \leftarrow \text{false}$ 
6: while  $t \leq \text{obt}$  do
7:    $C = \{\{s_i\}_{i=1}^{i=t}\} \leftarrow \text{GetCombinations}(E, t)$ 
8:   for  $c \in C$  do
9:      $\kappa_{\text{obt}} \leftarrow \text{Interpolate}(c)$ 
10:    if  $H(\kappa_{\text{obt}}) = h$  then
11:       $\kappa_{\text{rec}} \leftarrow \kappa_{\text{obt}}$ 
12:       $\text{recovered} \leftarrow \text{true}$ 
13:    break
14:  if  $\text{recovered} = \text{true}$  then
15:    break
16:   $t \leftarrow t + 1$ 

```

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$$(1 - (1 - \tau)\tau_j) \lfloor \delta_j n_T \rfloor < \tau \lfloor \delta_j n_T \rfloor$$

and hence

$$\sum_{j=i+1}^{\gamma} \lfloor \delta_j n_T \rfloor ((\tau-1)\tau_j + 1) < \sum_{j=i+1}^{\gamma} \tau \lfloor \delta_j n_T \rfloor. \quad (34)$$

Adding Eqs. (33) and (34) and  $\tau \lfloor \delta_i n_T \rfloor$  to both sides, we have

$$\begin{aligned} \tau \sum_{j=1}^{i-1} \lfloor \delta_j n_T \rfloor \tau_j + \tau \lfloor \delta_i n_T \rfloor + \sum_{j=i+1}^{\gamma} \lfloor \delta_j n_T \rfloor ((\tau-1)(\tau_j + 1)) \\ < \sum_{j=1}^{i-1} \tau \lfloor \delta_j n_T \rfloor + \tau \lfloor \delta_i n_T \rfloor + \sum_{j=i+1}^{\gamma} \tau \lfloor \delta_j n_T \rfloor \\ = \tau n_T. \end{aligned} \quad (35)$$

Therefore,  $n_{T,\text{rec}}$  can take values less than  $\tau n_T$ . A way of ensuring  $n_{T,\text{rec}}$  takes values only greater than  $\tau n_T$  is by setting  $\delta_j = 0, \forall j \in \{1, \dots, \gamma\} \setminus \{i\}$  and  $\delta_i = 1$ , which implies a uniform distribution.

In Algorithm 2, we present the reconstruction algorithm for the single-layered approach.

### 3. Computation Complexity

Consider the scenario where a user has to contact up to  $t$  trustees out of  $n$  contacts in her address book to recover her secret key. The number of computations NC in this case is given by the expression

$$\text{NC} = \sum_{i=2}^t \binom{n}{i}. \quad (36)$$

As shown in Eq. (36), the number of computations that  $\mathcal{U}$  needs to make in order to recover  $\kappa$  is a combinatorial object. To study the complexity, we derive standard lower

and upper bounds for  $\binom{n}{i}$ , where  $i \leq n$ , and use them to obtain bounds on NC.

**Lower bound to  $\binom{n}{i}$ :**

Let  $0 < m < i \leq n$ , then a simple computation gives  $\frac{n}{i} \leq \frac{n-m}{i-m}$ , as  $i \leq n$  implies the following equivalences:

$$\begin{aligned} 1 - \frac{m}{i} &\leq 1 - \frac{m}{n} \\ \Leftrightarrow \frac{i-m}{i} &\leq \frac{n-m}{n} \\ \Leftrightarrow \frac{n}{i} &\leq \frac{n-m}{i-m}, \end{aligned} \quad (37)$$

with the inequality strict unless  $n = i$ . Using Eq. (37), we can lower bound  $\binom{n}{i}$  as

$$\begin{aligned} \binom{n}{i} &= \frac{n}{i} \times \frac{n-1}{i-1} \times \dots \times \frac{n-(i-1)}{1} \\ &> \frac{n}{i} \times \frac{n}{i} \times \dots \times \frac{n}{i} \\ &= \left(\frac{n}{i}\right)^i. \end{aligned} \quad (38)$$

**Upper bound to  $\binom{n}{i}$ :**

Similarly, we can upper bound  $\binom{n}{i}$  as

$$\begin{aligned} \binom{n}{i} &= \frac{n!}{(n-i)! i!} \\ &= \frac{n \times (n-1) \times \dots \times (n-(i-1))}{i!} \\ &\leq \frac{n^i}{i!}. \end{aligned} \quad (39)$$

The power series expansion of  $e^k$  for real  $k$  gives us the relation  $e^i > \frac{i^i}{i!}$ , since

$$e^k = \sum_{i=0}^{\infty} \frac{k^i}{i!}$$

for any  $k$ , in particular for  $k = i$ . This leads to the equivalences

$$\begin{aligned} e^i &> \frac{i^i}{i!} \\ \Leftrightarrow \frac{1}{i!} &< \left(\frac{e}{i}\right)^i \\ \Leftrightarrow \frac{n^i}{i!} &< \left(\frac{ne}{i}\right)^i. \end{aligned} \quad (40)$$

Combined with Eq. (39), we have an upper bound, given by

$$\binom{n}{i} < \left(\frac{ne}{i}\right)^i. \quad (41)$$

Therefore, from Eq. (38) and Eq. (41), we can bound the number of computations by

$$\sum_{i=2}^t \left(\frac{n}{i}\right)^i < \text{NC} < \sum_{i=2}^t \left(\frac{ne}{i}\right)^i. \quad (42)$$

To obtain closed form expressions that can be interpreted, we loosen the bounds in Eq. (42) to

$$\sum_{i=2}^t \left(\frac{n}{2}\right)^i < \text{NC} < \sum_{i=2}^t \left(\frac{ne}{2}\right)^i, \quad (43)$$

by simply lower and upper bounding  $i$  by 2 and  $t$  respectively. Both bounds are now in the form of a simple geometric series, and can be evaluated in closed form using the formula  $a + ar + \dots + ar^{n-1} = a \frac{r^n - 1}{r - 1}$  for  $r \neq 1$ . This gives us the bounds

$$\frac{n^2}{t(n-t)} \left( \left(\frac{n}{2}\right)^{t-1} - 1 \right) < \text{NC} < \frac{(ne)^2}{2(ne-2)} \left( \left(\frac{ne}{2}\right)^{t-1} - 1 \right). \quad (44)$$

Hence, for a constant  $t$ , the value NC grows polynomially with  $n$ . On the other hand, for a constant  $n$ , this value grows exponentially with increase in  $t$  (as long as  $t \leq n$ ).

Fig. 17 depicts the wallclock time taken per reconnection and Fig. 16 shows the wallclock time for total computation. As mentioned earlier, the trends of these plots are essentially the same as the plots with CPU time but the values are different because of the parallelization of the implementation.

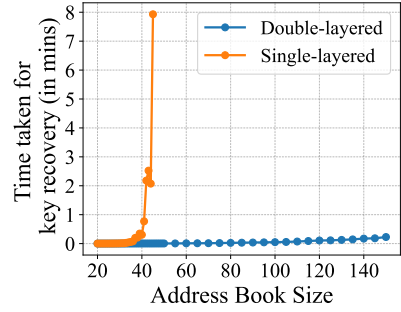


Figure 16. Wallclock time: total recovery time.

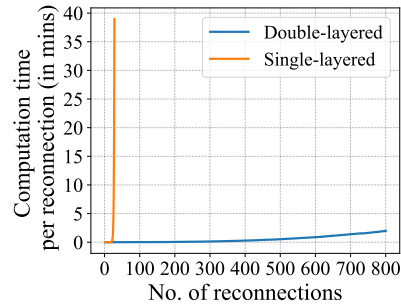


Figure 17. Wallclock time: time taken per reconnection.

Fig. 18 depicts the variation of probability of user recovering her vault's key with different system parameters. As depicted in Fig. 18a, for having an 80% chance of succeeding, the number of people that need to be contacted increases with the address book size. The two-layered setting requires significantly more reconnections as compared to the

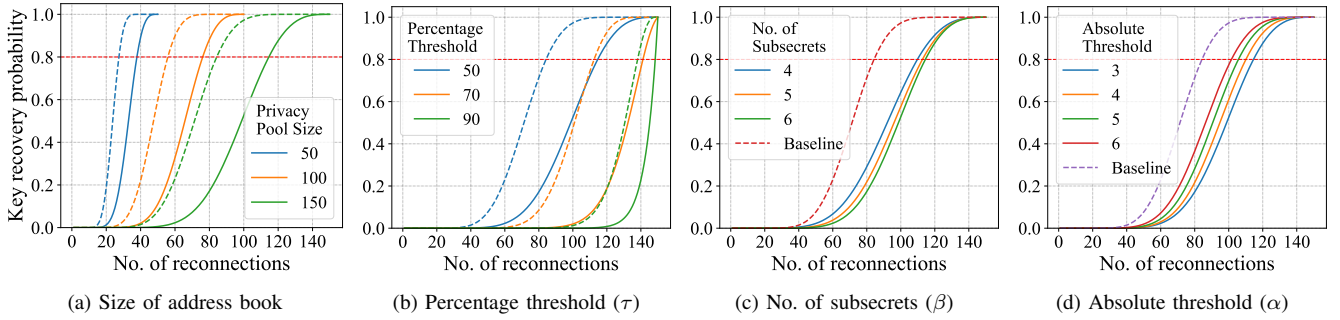


Figure 18. Probability of secret recovery: for (a) and (b), dashed curves correspond to the single-layered approach for the solid curve; for (c) and (d), dashed curves correspond to single-layered approach with parameters in Table 3

single-layered setting due to the probabilistic key recovery. On one hand, it demands more effort from the user, while on the other it makes coercive recovery more difficult for an adversary. Fig. 18b shows that for a given probability, the number of people, that need to be approached for key recovery, increases with percentage threshold and the two-layered approach again requires more reconnections. Since packets need to be obtained from more trustees, a coercive adversary needs to coerce more people for succeeding. This is because the coercer needs to coerce a larger number of people to obtain the key. Fig. 18c depicts that for a given probability, the number of contacts that need to be approached increases with the number of subsecrets. We observe such a trend because one needs to reconstruct more subsecrets to recover the key and in turn,  $\mathcal{A}$  needs to coerce a larger number of people to reconstruct each subsecret for the key recovery. This result backs our design choice made in Section 5.4. Finally, with the increase in the absolute threshold, there are two parameters that have opposing impact on the probability: (i) increase in the no. of shares needed to reconstruct a subsecret; (ii) decrease in the no. of subsecrets (since we keep the no. of shares held by a person ( $\gamma$ ) constant). As depicted in Fig. 18d, as the absolute threshold is increased, the latter dominates the former’s impact for larger thresholds, thereby making the system less secure. This section presents three extensions of Apollo: Apollo Lite, Thresholded-Apollo and Hinted-Apollo that make the recovery process easier for  $\mathcal{U}$ . In MLSS,  $\mathcal{U}$  generally has to contact more than threshold fraction of her trustees to reconstruct  $\kappa$ , thus demanding more effort from  $\mathcal{U}$ ’s end. Apollo Lite reduces the threshold of each subsecret by increasing the number of shares per subsecret. This increases the number of relevant shares distributed and brings the probability variation closer to that of the single-layered approach. On the other hand, Thresholded-Apollo or T-Apollo uses threshold in the subsecrets layer to reduce the number of contacts approached. Hinted-Apollo or H-Apollo adds encrypted hints to the blobs to make the approach of contacts less arbitrary. Next, we describe the design of these variants, and also elaborate on their impact of security.

#### 4. Apollo Lite

This extension is for shifting the expected probability of recovery to that of a single-layered approach. Instead of defining the minimum number of trustees that should be contacted, such users define the threshold in terms of the expected number of trustees that should be contacted for recovering the key,  $\kappa$ . For this purpose, the threshold of the subsecrets needs to be reduced and in Apollo, we can achieve this by increasing the number of shares per subsecret. On one hand, this would increase the probability of key recovery at the threshold set by the user, while on the other, it also introduces cases where the key can be recovered by contacting less than threshold number of people. Therefore, we analyze the impact of modifying the share generation for bringing the expected threshold closer to the threshold defined by  $\mathcal{U}$ . As depicted by Fig. 19, the amount of effort from user reduces as the number of extra shares increases.

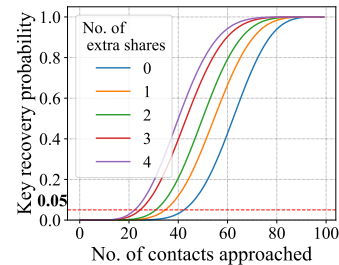


Figure 19. Reduction in the no. of connections needed

#### 5. T-Apollo

Thresholded-Apollo or T-Apollo uses a modified version of MLSS, which we refer to as thresholded-MLSS or TMLSS. The design of TMLSS is similar to that of MLSS, except the fact that TMLSS uses Shamir’s secret shares in the subsecrets layer. Using a threshold for subsecrets is not equivalent to using a smaller number of subsecrets since the former provides higher share availability by introducing redundancy in the shares distributed. Thus, such a

$(x, y)$	salt	$\text{Enc}_{y_{ss}}(\text{salt}    00 \dots 00    x_{ss})$	$\text{Enc}_{\kappa}(\text{salt}    00 \dots 00    0)$
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Figure 20. Structure of a blob in Thresholded-MLSS

design serves two main purposes in key recovery. Firstly, it increases the probability of key recovery after contacting  $\lceil \tau n_T \rceil$  people, thus ameliorating the issue of non-recovery in MLSS (Fig. 13). Secondly, TMLSS makes key recovery possible in the scenario where people in the address book are not responsive. On the downside, TMLSS increases the probability of adversarially recovering the key via coercion as compared to the design using additive secret sharing in subsecrets (MLSS). Now, we proceed to explain the design of TMLSS.

**5.1. Share distribution.** The share generation algorithm of TMLSS is similar to that of MLSS 5.4, with the only difference in the way subsecrets are generated. In TMLSS, subsecrets are Shamir’s secret shares of  $\kappa$  with the percentage threshold ( $\tau_S$ ). The value of  $\tau_S$  can either be a default system value, or it can be set based on  $\mathcal{U}$ ’s expected threshold. Thus, in TMLSS, each subsecret consists of a pair,  $\kappa_l = (x_l, y_l)$ , s.t.,  $x_l \xleftarrow{R} \mathbb{Z}, y_l = \phi(x_l) \in \mathbb{Z}_p$ . For each subsecret  $\kappa_l$ , Shamir’s secret shares are generated, such that each share is  $\kappa_{l,m} = (x_{l,m}, y_{l,m})$ , s.t.,  $x_{l,m} \xleftarrow{R} \mathbb{Z}, y_{l,m} = \phi_l(x_{l,m}) \in \mathbb{Z}_p$ , where  $\phi_l$  is a polynomial with constant term  $y_l$  (y-coordinate of the subsecret,  $\kappa_{l,m}$ ), and these shares are distributed among the trustees. The structure of random shares remains the same as in the case of MLSS:  $(x', y')$ , s.t.,  $x' \xleftarrow{R} \mathbb{Z}, y' \xleftarrow{R} \mathbb{Z}_p \setminus \{\phi_l(x') | \forall l \in \{1, \dots, \beta\}\}$ . Furthermore, the method used for assigning key shares and random shares among trustees for maintaining indistinguishability remains the same as MLSS Section 5.4.

**5.2. Packet structure.** In TMLSS,  $\kappa_l$  consists of an x-coordinate ( $x_l$ ), and a y-coordinate ( $y_l$ ). While the share distribution backs up Shamir’s secret shares of  $y_l$ , it does not include any information about  $x_l$ . Thus, a blob needs to store the value of  $x_l$ , while maintaining the indistinguishability of blobs. Storing  $x_l$  in plaintext would imply that two packets with shares of the same subsecret will have the same  $x_l$ , enabling  $\mathcal{A}$  to tell if two packets correspond to the same subsecret. Since blobs in TMLSS need to store some information instead of only indicating successful recovery like MLSS, hashing relevant information for recovery would not suffice. Thus, TMLSS encrypts the tag and additional information relevant for key recovery. The encryption should store  $x_l$  corresponding to the recovered  $y_l$ , while also acting as an indicator for successful recovery (tag). If packets simply store the encryption of  $x_l$  along with some indicator bits,  $\mathcal{A}$  can figure out if two packets contain shares of the same subsecret as AES encryption is not randomized. To address this issue, blobs include a salt to randomize the encryption. For indicating successful decryption of a subsecret  $\kappa_l = (x_l, y_l)$ , TMLSS pads the salt and some 0’s to  $x_l$ .

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### Algorithm 3 Reconstruct<sub>TMSS</sub>

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1: Input:  $P_{\text{obt}} = \{\text{pkt}_i\}_{i=1}^k, \alpha$ 
2: Output:  $\kappa_{\text{rec}} \in \{\kappa, \perp\}$ 
3:  $K = \{\kappa_i\} \leftarrow \text{ExtractShares}(P_{\text{obt}})$ 
4:  $E \leftarrow \text{GetEncryptionDict}(P_{\text{obt}})$ 
5:  $R \leftarrow \text{GetNonceDict}(P_{\text{obt}})$ 
6:  $\kappa_{\text{rec}} \leftarrow \perp, \text{recovered} \leftarrow \text{false}, S \leftarrow []$ 
7:  $C = \{\{\kappa_i\}_{i=1}^\alpha\} \leftarrow \text{GetCombinations}(K, \alpha)$ 
8: for  $c \in C$  do
9:    $y_{\text{int}} \leftarrow \text{Interpolate}(c)$ 
10:   $E_{\text{relevant}} \leftarrow E[c[0]], \text{salt} \leftarrow R[c[0]], \kappa_{\text{obt}} \leftarrow \perp$ 
11:  for  $e \in E_{\text{relevant}}$  do
12:     $p = \text{Dec}_{\text{AES}}(y_{\text{int}}, e)$ 
13:    if  $\text{StartsWith}(p, \text{salt} || 00 \dots 00)$  then
14:       $x_{\text{int}} = \text{GetXCoordinate}(\text{plaintext})$ 
15:       $\text{ss}_{\text{int}} \leftarrow (x_{\text{int}}, y_{\text{int}})$ 
16:      if  $\text{ss}_{\text{int}} \notin S$  then
17:         $\text{append}(S, \text{ss}_{\text{int}})$ 
18:         $\kappa_{\text{obt}} = \text{Interpolate}(S)$ 
19:      if  $\kappa_{\text{obt}} \neq \perp$  and  $\text{len}(S) \neq 1$  then
20:        for  $e \in E_{\text{relevant}}$  do
21:           $p = \text{Dec}_{\text{AES}}(\kappa_{\text{obt}}, e)$ 
22:          if  $\text{StartsWith}(p, \text{salt} || 00 \dots 00)$  then
23:             $\kappa_{\text{rec}} \leftarrow \kappa_{\text{obt}}, \text{recovered} \leftarrow \text{true}$ 
24:            break
25:          if  $\text{recovered} = \text{true}$  then
26:            break
27: return  $\kappa_{\text{rec}}$ 

```

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For storing the indicator of recovery of  $\kappa$ , TMLSS considers the x-coordinate of the secret key as 0. Thus, blob includes, (i)  $e_l = \text{Enc}_{y_l}(\text{salt} || 00 \dots 00 || x_l)$  for every key share  $\kappa_{l,m}$  in the packet; and (ii)  $e_\kappa = \text{Enc}_\kappa(\text{salt} || 00 \dots 00 || 0)$ . The structure of a blob has been depicted in Fig. 20. For random shares, rather than storing salted encryptions, blob contains two random strings of length same as that of  $e_l$ . Similar to MLSS, for  $b_i \in B_T$ ,  $\text{pkt}_i = \{\text{blob}\}$ . Assuming that the deployed encryption algorithm is semantically secure, each blob would be indistinguishable from the other, thus making the trustees indistinguishable from the non-trustees.

**5.3. Key Reconstruction.** The key reconstruction algorithm of TMLSS briefly has been depicted in Algorithm 3. Just like MLSS, TMLSS does not expect the user to remember her set of trustees, or the thresholds used. The key reconstruction in TMLSS is similar to MLSS, only differing in the checksum verification and the reconstruction of  $\kappa$  from subsecrets. Specifically, instead of matching salted hashes of the subsecrets,  $\mathcal{U}$  decrypts the encryptions of the blobs to detect correct recovery. If the algorithm detects the blob’s salt and some padded zeros in the decryption, then the algorithm recognizes successful recovery of the subsecret and obtains the corresponding  $x_l$  from the decryption. Additionally, since subsecrets are Shamir’s secret shares of  $\kappa$ , TMLSS attempts to reconstruct  $\kappa$  from the subsecrets by using polynomial interpolation (instead of adding them).



$(x, y)$	salt	$\text{Enc}_{ss}(\text{salt}    00 \dots 00    \text{hint})$	$\text{Enc}_{\kappa}(\text{salt}    00 \dots 00    0)$
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Figure 21. Structure of a blob in Hinted-MLSS

**5.4. Security Implications.** With T-Apollo,  $\mathcal{U}$  needs to reconstruct lesser subsecrets with the same number of people holding the shares as in the case of MLSS. Since one needs to obtain  $\tau_S$  fraction of the total shares for reconstructing  $\kappa$ , the minimum number of trustees that need to be contacted also drops by  $\tau_S$ . On one hand, TMLSS reduces the amount of effort needed from  $\mathcal{U}$ 's end for reconstruction, while on the other, it increases the probability of coercive recovery as compared to MLSS. We evaluate and discuss this trade-off in detail later.

With T-Apollo described, we now explain the design of H-Apollo.

## 6. H-Apollo

Hinted-Apollo or H-Apollo modifies the way key recovery works in Apollo using the recovery address book. Particularly, H-Apollo stores some *hints* in the blobs held by trustees that provide the name of trustees holding shares of another subsecret. The hints are encrypted using subsecrets as the trapdoor, thereby guiding  $\mathcal{U}$  as she proceeds with the recovery. Thus, H-Apollo reduces the amount of effort required from  $\mathcal{U}$ 's end for obtaining shares from her address book. Consequently, it increases the probability of secret recovery after contacting  $\lceil \tau n_T \rceil$  trustees. Conversely, it could provide  $\mathcal{A}$  with hints during coercive recovery, leading to weaker security as compared to Apollo. H-Apollo uses HMLSS, a slightly modified version of MLSS, which we describe next.

**6.1. Share distribution.** The share generation in HMLSS is similar to that in MLSS. The subsecrets are additive shares of  $\kappa$ , and the key shares are the Shamir's secret shares of the subsecrets.

**6.2. Packet structure.** In H-Apollo, the hints are revealed as  $\mathcal{U}$  reconstructs more subsecrets. Hence, the hints are encrypted and stored in the blobs held by trustees using subsecrets as encryption keys. The packet structure in HMLSS is same as in the case of TMLSS, with the only difference in the content of the encryption — instead of storing  $x$ -coordinates, blobs store names of the trustees. Similarly, for the encryption corresponding to  $\kappa$ , the blobs store the encryption of a string of 0's, instead of a hint. A blob's structure in H-Apollo is depicted in Fig. 21.

Another parameter that governs the security of H-Apollo is the number of hints stored across all the blobs. If we store hints naively and back up names of all the trustees across all the blobs, then it would increase the chances of successful coercive recovery because recovering one subsecret would lead to rest of the trustees. Therefore, H-Apollo places a

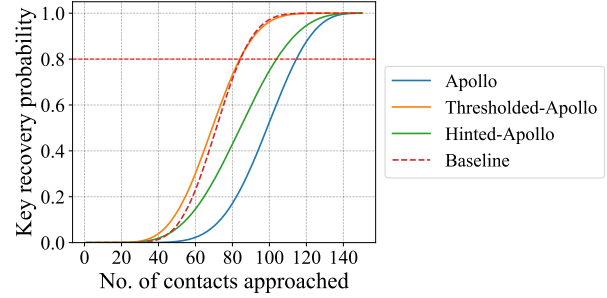


Figure 22. Comparison with variants (probability of key recovery): TMLSS with 80% threshold in the subsecrets and HMLSS with 5 hints

bar on the number of trustees revealed by the blobs as a whole. For example, consider only one trustee has to be hinted by the address book. Here, all the blobs will store the encryption of exactly one trustee. When multiple hints have to be provided by all the blobs, the names of trustees are sampled uniformly randomly from a subset of names. That is, when the number of hints is chosen to be  $n_{\text{hints}}$ , then H-Apollo chooses the names of trustees to be hinted:  $B_{\text{hints}} \stackrel{R}{\leftarrow} B$ , such that  $|B_{\text{hints}}| = n_{\text{hints}}$ . For each blob containing a key share, the hint is chosen as:  $\text{hint} \stackrel{R}{\leftarrow} B_{\text{hints}}$ . Based on  $\mathcal{U}$ 's requirements for expected threshold, H-Apollo can set the number of trustees to be revealed by all the blobs ( $n_{\text{hints}}$ ). For random shares, the blobs do not store any hints, and instead store two random strings of length same as the encryption in blobs containing key shares.

**6.3. Key Reconstruction.** The key reconstruction algorithm in HMLSS is identical to that in TMLSS (Algorithm 3). The difference lies in the way  $\mathcal{U}$  approaches people in her address book and in the way,  $\kappa$  is reconstructed from subsecrets.  $\mathcal{U}$  uses hints in the blobs to make the overall key recovery process relatively less arbitrary. Since she approaches more trustees as she starts recovering subsecrets, the overall computation time also reduces with respect to MLSS. Moreover, since the subsecrets are additive shares of  $\kappa$ ,  $\kappa$  is reconstructed by adding the subsecrets.

**6.4. Security Implications.** In H-Apollo,  $\mathcal{U}$  is provided with hints, such that she needs to contact a lesser number of people. While it makes recovery easier for  $\mathcal{U}$ , H-Apollo also leaks more information to  $\mathcal{A}$  as compared to Apollo. We analyze these implications in detail later.

Having described Apollo and its variants, we now illustrate the various practical aspects involved in designing Apollo.

## 7. Comparison with Apollo

In Fig. 22, we compare the probability of key recovery using Apollo and the variants described above.